Cognitive Development 59 (2021) 101083

Contents lists available at ScienceDirect

Cognitive Development

journal homepage: www.elsevier.com/locate/cogdev

Foundations for early mathematics skills: The interplay of approximate number system, mapping ability, and home numeracy activities

Liyue Guo [a,b,1], Xinpei Xu [c,1], David Yun Dai [d], Ciping Deng [a,b,*]

[a] Shanghai Key Laboratory of Brain Functional Genomics, Affiliated Mental Health Centre (ECNU), School of Psychology and Cognitive Science, East China Normal University, Shanghai, 200062, China
[b] Shanghai Changning Mental Health Center, Shanghai, China
[c] Shanghai Early Childhood Education College, Shanghai Normal University, 200234, China
[d] Department of Educational & Counseling Psychology, School of Education, State University of New York-Albany, Albany, New York, 12222, USA

ARTICLE INFO

Keywords:
Mathematics skills
Approximate number system
Mapping ability
Home numeracy activities

ABSTRACT

This study investigated the conceptual framework of how approximate number system (ANS), mapping ability, and home numeracy activities together contribute to the development of children's early mathematics skills. Participants were 110 children (58 boys) and their mothers. Children were individually tested on ANS, and their mothers completed the questionnaire of home numeracy activities. One year later, children were administrated the test of intelligence, a test of mapping ability, and the Test of Early Mathematical Ability-3 (TEMA-3). Results indicated that ANS and home numeracy activities directly predicted informal mathematics skills and indirectly predicted formal mathematics skills through mapping ability.

1. Introduction

Children's early mathematics skills provide the foundation for later mathematics learning (e.g., Rittle-Johnson, Fyfe, Hofer, & Farran, 2017; Träff, Olsson, Skagerlund, & Östergren, 2020). Empirical evidence showed that approximate number system (ANS), mapping ability, home numeracy activities, and intelligence play an important role in the development of children's early mathematics skills (Huang, Zhang, Liu, Yang, & Song, 2017; Schneider et al., 2017; Wong, Ho, & Tang, 2016). This fact has led to the emergence of new lines of research to examine how these factors, involving nature and nurture, jointly contribute to early mathematics skills. Previous studies have indicated that parental variables (e.g., parent attitude and beliefs about early mathematics) combined with numeracy-related activities predict children's numeracy scores (e.g., Skwarchunk, 2009); but we know little about how child variables (e.g., ANS and mapping ability) and numeracy-related activities work together to improve children's mathematics skills. Mapping ability, the process of associating number symbols with corresponding magnitudes, has been found to mediate the relationship between ANS and early mathematics skills (e.g., Jang & Cho, 2018; Rittle-Johnson et al., 2017; Wong et al., 2016). In addition, mapping

* Corresponding author at: Shanghai Key Laboratory of Brain Functional Genomics, Shanghai Changning-ECNU Mental Health Center, School of Psychology and Cognitive Science, East China Normal University (ECNU), 3663 North Zhongshan Road, Shanghai, China.
E-mail addresses: liyueguo27@126.com (L. Guo), xpxu@shnu.edu.cn (X. Xu), ydai@albany.edu (D.Y. Dai), cpdeng@psy.ecnu.edu.cn, cpdeng@gmail.com (C. Deng).

[1] Liyue Guo and Xinpei Xu contributed equally to this work.

https://doi.org/10.1016/j.cogdev.2021.101083
Received 24 November 2020; Received in revised form 22 June 2021; Accepted 29 June 2021
Available online 2 July 2021

ability has been found to be positively associated with home numeracy activities (Mutaf Yildiz, Sasanguie, Smedt, & Reynvoet, 2018; Susperreguy, Douglas, Xu, Molina-Rojas, & LeFevre, 2020) and mathematics skills (Mundy & Gilmore, 2009; Jiménez Lira, Carver, Douglas, & LeFevre, 2017); mapping ability is also likely to be a mediator of relationships between home numeracy activities and mathematics skills (Wei, Li, & Su, 2020). However, since this body of research has generally been conducted in Western societies, it is unclear as to whether this set of complex relationships holds in Chinese children. Thus, the present study aimed to examine whether mapping ability serves as a mediator of relations between ANS and early mathematics skills as well as relations between home numeracy activities and mathematics skills in Chinese preschool children.

1.1. ANS, mapping ability, and mathematics skills

ANS refers to the ability to represent large, approximate non-symbolic numerical magnitudes (Feigenson, Dehaene, & Spelke, 2004). Research have firmly confirmed the positive association between ANS and mathematics skills, showing correlations ranging from 0.20 to 0.28 (Chen & Li, 2014; Fazio, Bailey, Thompson, & Siegler, 2014; Schneider et al., 2017), with the strongest association in preschool (Fazio et al., 2014). ANS and symbolic representations have common structures and functions, such as successor principle, which provides cardinal meanings (Gallistel & Gelman, 1992). Children understand the cardinal values of symbolic numerals by mapping them onto ANS (vanMarle et al., 2018).

In the model of mathematical leaning (Geary, 2013), mapping ability was suggested to be a mediator of the association between ANS and mathematics skills. Geary (2013) asserted that formal mathematics skills are mastered by three steps: in the first step, children achieve the ability to discriminate quantities, mainly through ANS; in the second step, children master the meaning of number words and Arabic numerals through linking them with their represented quantities; in the third step, children are able to explicitly understand the logical structure of the number system, such as the relations among numbers. In line with this model, mapping ability is an intermediary factor in the associations between ANS and mathematics skills, supported by many empirical studies with preschool children, elementary school students, and adults (e.g., Jang & Cho, 2018; Rittle-Johnson et al., 2017; Wong et al., 2016). However, these studies did not distinguish between formal and informal mathematics skills. Formal mathematics skills refer to the ability to represent and manipulate quantities by the symbolic number system, such as number words and Arabic numerals; in contrast, informal mathematics skills refer to mathematics skills involving the non-symbolic numerical magnitudes, such as object-based operations (Skwarchuk, Sowinski, & LeFevre, 2014; Zhang, Hu, Zou, & Ren, 2020). Informal mathematics skills are precursors to formal mathematics skills (Purpura, Baroody, & Lonigan, 2013). In addition, ANS was more related to informal mathematics skills than formal mathematics skills (Libertus, Feigenson, & Halberda, 2013; Libertus, Odic, Feigenson, & Halberda, 2016), because the completion of formal mathematics tasks for children is contingent on the development of mapping ability (Geary, 2013; Wong et al., 2016). In this study, we distinguished between these two types of mathematics skills and examined whether and how mapping ability mediates the relations between ANS and the two types of mathematics skills.

1.2. Home numeracy activities, mapping ability and mathematics skills

The concept of home numeracy activities is indicative of parental involvement in children's math-related learning that contribute to children's mastery of mathematics knowledge and skills. It is a two-dimensional construct that comprises formal and informal home numeracy activities (Huang et al., 2017; LeFevre et al., 2009; Skwarchuk et al., 2014). Formal home numeracy activities are direct and intentional parent-child teaching activities at home on the topic of numeracy, including activities of number skills and number books. In contrast, informal home numeracy activities are indirect and implicit teaching activities, involving activities of games and applications. These activities are thought to be crucial for the child's formal and informal mathematics skills at preschool age.

According to the home numeracy model (Skwarchuk et al., 2014), formal home numeracy activities have an intentional focus on symbolic number system; these activities are meant to develop formal mathematics skills. In contrast, informal home numeracy activities involve little knowledge on the symbolic number system; these activities help inculcate informal mathematics skills. Empirically, however, findings are not as clear regarding the different types of home numeracy activities and formal versus informal mathematics skills in preschool children. Some studies found positive associations between number skills activities and formal mathematics skills, between game activities and informal mathematics skills (Skwarchuk et al., 2014; Susperreguy, Di Lonardo Burr, Xu, Douglas, & LeFevre, 2020). Other studies found positive associations between number skills activities and informal mathematics skills (Gunderson & Levine, 2011), between informal learning activities involving games or applications and formal mathematics skills (LeFevre et al., 2009; Zhang et al., 2020). Still others found no relationship between home numeracy activities and mathematics skills in children aged 3–6 years (Blevins-Knabe, Austin, Musun, Eddy, & Jones, 2000; Missall, Hojnoski, Caskie, & Repasky, 2015; Zippert, Douglas, Smith, & Rittle-Johnson, 2020). These inconsistent results may be due to the different measures of home numeracy activities. Studies that assessed parents' math support within a play session during laboratory observation may over- or under-estimate children's early home math experiences with parents. Indeed, laboratory observations of parents' math support do not correlate significantly with parental reports in past studies (Missall, Hojnoski, & Moreano, 2017; Mutaf Yildiz, Sasanguie, Smedt, & Reynvoet, 2018). In comparison, observations or parental reports on the frequency of home numeracy activities in natural home settings may result in more consistent conclusions (Missall et al., 2017); and parent report is more convenient and commonly used than real situation observation. Thus, in the present study, we measured home numeracy activities through parent-report. In addition, these inconsistent results indicate that the necessary to explore the potential mechanism of home numeracy activities affecting children's mathematics skills.

Home numeracy activities, as one kind of leaning experiences for children, were found to be correlated with mapping ability (Mutaf Yildiz et al., 2018a; Susperreguy, Di Lonardo Burr et al., 2020). In formal home numeracy activities, children may practice to transcode between non-symbolic quantities and symbolic numerals and improve their automatic processing of number symbols, hence a strong foundation for learning higher mathematics skills (Kolkman, Kroesbergen, & Leseman, 2013). In informal home numeracy activities, children's experience with concrete non-symbolic mathematics problems may facilitate abstraction of the embedded rule or principle, which, in turn, can be applied to symbolic mathematics problems (Sherman & Bisanz, 2009). The concrete objects in these contexts may also help children to build stronger associations between symbolic numerals and non-symbolic quantities (Zhang et al., 2020). Taken together, in both types of activities, children have the opportunity to practice and acquire mapping ability, which is useful for improving their mathematics skills. However, for different types of mathematics skills, the importance of mapping ability may vary. As mentioned earlier, mapping ability may have a prominent role in formal mathematics skills (Libertus et al., 2013, 2016). Thus, in the present study, we examined whether and how mapping ability would mediate the relationship between home numeracy activities and children's formal and informal mathematics skills.

1.3. The present study

There is intensive evidence in Western and Chinese samples indicating that ANS and home numeracy activities are related to mathematics skills in children and that mapping ability mediates this relationship between ANS and mathematics skills. Compared with American parents, Chinese parents tend to use more directives and commands in assisting children during preschool years (Huntsinger & Jose, 2009; Huntsinger, Jose, Liaw, & Ching, 1997). Home numeracy activities may work together with ANS to play a crucial role in the development of children's mapping ability, which in turn promote children's early mathematics skill. However, to our knowledge, no study that had been carried out with children included the aforementioned four constructs; also, no study simultaneously examined the mediating role of mapping ability on the relationship between ANS and mathematics skills, as well as the relationship between home numeracy activities and mathematics skills. In view of the developmental nature of mathematics skills, it is important to explore the foundations for mathematics skills. For this reason, the main goals of this study were (a) to investigate correlations of ANS, mapping ability, and home numeracy activities (number skills, number books, games, and applications) with formal and informal mathematics skills, (b) to test a structural equation model that includes the relationships between the predictor variables (i.e., ANS, mapping ability, and home numeracy activities) and the criterion ones (i.e., formal and informal mathematics skills). This technique allowed us to simultaneously test all relations among predictors and criterion variables in the model, with the model free of measurement errors. Given the previous theoretical predictions and empirical evidence, we postulated the hypotheses as follows:

(1) ANS, mapping ability, and home numeracy activities are related to the two types of mathematics skills, formal and informal (Hypothesis 1), and (2) the relationship between ANS and mathematics skills as well as the relationship between home numeracy activities and mathematics skills are mediated by individual differences in mapping ability, especially with respect to the formal rather than informal kind of mathematics skills (Hypothesis 2). Given that intelligence was a steady predictor of mathematics skills (Wei et al., 2020; Wong et al., 2016), in the present study, we statistically controlled the effects of intelligence on the two types of mathematics skills when running the structural equation model.

2. Method

2.1. Participants

Participants were recruited from two public kindergartens in Shanghai, China. In China, formal mathematics curriculum was not provided in kindergartens (Li, McFadden, & DeBey, 2019; Liu, Zhang, Song, & Yang, 2019). Data were collected at two points in time, in October and November of 2018 (T1; second year of kindergarten), and October and November of 2019 (T2; third/final year of kindergarten). At T1, 110 children (58 boys and 52 girls; age: $M = 54.96$ months, $SD = 3.64$ months) and their mothers participated. Children were native Mandarin speakers. Yearly family income was measured on a 7-point scale ranging from 1 (30,000 RMB or below) to 7 (above 500,000 RMB). Both the median and the mode of yearly family income were between 200,000 and 500,000 RMB, which was representative of the average yearly income of the general population in the region (Shanghai Statistical Yearbook, 2019). The majority of the children were from middle socioeconomic backgrounds. A total of 108 mothers reported their education and home numeracy activities with their children. Of the mothers, 6.48 % had a middle school education, 6.48 % had high school education, 8.33 % attended vocational school, 35.18 % held a college degree, 41.67 % held graduate degree, and 1.85 % held a doctoral degree. At T2 ($N = 102$), 8 children (1 boy, 7 girls) did not participate because these children were absent (3 children) or transferred to other schools (5 children). A comparison was made between the present and absent children. The results showed that these two groups had no significant differences in all of the variables measured at T1, $F(6, 94) = 0.20$, $p > .05$. The full information maximum-likelihood (FIML) method was used to deal with the missing data of those who did not participate at both times of the data collection. Thus, following analyses were performed with the data of 110 children and their mothers.

2.2. Materials

2.2.1. Approximate number system

The Panamath (http://www.panamath.org/) was adopted to measure children' ANS accuracy. Prior research has used the measure in the Chinese context (e.g., Peng, Yang, & Meng, 2017). In the task, participants were presented with two sets of dots (blue dots and yellow dots) at each trial and children indicated which set was more numerous. The number of dots in each set varied from 5 to 21. A total of 2 practice trials and 96 experimental trials were displayed and the ratio of blue and yellow dots was varied between 1.25 and 3.17. Dots were displayed for 2322 ms, and children could go to next trial only after they made a response. The cumulative area of dots was positively correlated with numerosity in half of the trials and negatively correlated with numerosity in the other half. The Cronbach's alpha was 0.88.

2.2.2. Mapping ability

The mapping task was adapted from Mundy and Gilmore (2009), and presented using the E-prime software. At each trial, an Arabic symbol and two dot arrays were simultaneously presented. The target Arabic symbol involved the digits 5–28. The dot arrays varied from 5 to 46. On one half of the trials, the ratio between the correct dot arrays and the distracted dot arrays was 0.50. On the other half of the trials, the ratio between the correct dot arrays and the distracted dot arrays was 0.67. Children were instructed to match Arabic symbol to the corresponding dot arrays. A total of 4 practice trials and 24 experimental trials were administrated. The accuracy of mapping task was measured, with higher accuracy reflecting better mapping ability. The Cronbach's alpha was 0.70 for this test.

2.2.3. Home numeracy activities

Home numeracy activities were assessed using a mother-report measure tapping the frequency of mothers' engagement in numeracy activities (Huang et al., 2017). There were 21 items in the measure assessing four factors: number skills (10 items), number books (3 items), games (4 items), and applications (4 items). Mothers were asked to respond to the statements using a 5-point Likert scale (0 = *never occurred* to 4 = *almost daily*). A similar questionnaire has been used with Hong Kong Chinese children (Huang et al., 2017; Liu et al., 2019). We conducted confirmatory factor analyses to examine the factor structure of home numeracy activities in this present sample of (about 5-year-old) Chinese preschoolers in Shanghai. As suggested by previous research (e.g., LeFevre et al., 2009), two items of games (i.e., "making collections", "playing card games") and two items of applications (i.e., "playing with calculators", "having your child wear a watch") were removed because very few parents were reported engagement in these two activities. An evaluation of model fitting was undertaken by using the ratio chi-square over degrees of freedom (χ^2/df), the comparative fit index (CFI), the Tucker-Lewis fit index (TLI), the root-mean-square error of approximation (RMSEA), and the standardized root mean square residual (SRMR). If $\chi^2/df \leq 5$ and 3, CFI and TLI ≥ 0.90 and 0.95, and RMSEA and SRMR ≤ 0.08 and 0.06, model fitting is considered to be acceptable and excellent (Hu & Bentler, 1999). In general, if the vast majority of the indexes indicate an acceptable/excellent fit, then there is probably an acceptable/excellent fit. The four-factor model with the remaining 17 items showed a poor fit, $\chi^2 = 226.97$, $df = 113$, $p < .001$, RMSEA = 0.10, CFI = 0.84, TLI = 0.81, SRMR = 0.06. After adjusted the model by correlating the item "ordering things by size, length, weight, or number" with the item "discriminating different sizes and amounts", the model showed a somewhat better fit, $\chi^2 = 202.22$, $df = 112$, $p < .001$, RMSEA = 0.08, CFI = 0.88, TLI = 0.85, SRMR = 0.06. The means, standardized deviations, factor loadings of the remaining items are presented in Appendix A. The average score of responses within each factor was calculated, with higher scores indicating higher frequency of mothers' engagement in these four types of numeracy activities. The Cronbach's alphas were 0.88, 0.72, 0.60 and 0.58 for number skills, number books, games and applications. The low reliabilities for the games and applications may be due to the limited items in these two constructs.

2.2.4. Early mathematics skills

The Test of Early Mathematical Ability-3 (TEMA-3, Ginsburg & Baroody, 2003) has 72 items that measure children's early formal and informal mathematics knowledge. The formal conceptual categories include items targeting number literacy (e.g., "writing numerals"), number facts (e.g., "addition facts"), calculation (e.g., "written addition procedure"), and concepts (e.g., "symbolic additive commutativity"). The informal conceptual categories include items targeting numbering (e.g., "verbal counting by ones"), number comparisons (e.g., "choosing the larger number objects"), calculation (e.g., "nonverbal concrete addition and subtraction"), and concepts (e.g., "cardinality rule"). If children failed on five consecutive trials, the test would be discontinued. The correct number of trials of formal and informal mathematics skills were calculated, with higher scores reflecting better mathematics skills. Prior research has used the measure in the Chinese context (e.g., Wei et al., 2020). The Cronbach's alphas were 0.91 formal mathematics skills and 0.83 for informal mathematics skills, respectively.

2.2.5. Intelligence

Intelligence was assessed by nonverbal matrices from the Cognitive Assessment System-Version 2 (CAS-2, Naglieri & Das, 2014). The test consisted of 43 items. At each item, different shapes and geometric designs with a missing piece were shown, and children were asked to select one as the correct answer from the given five or six alternatives. Prior research has used the measure in the Chinese context (e.g., Wei et al., 2020). The correct number of items was used as the child's score, with higher score reflecting higher intelligence. The Cronbach's alpha on this test was 0.82.

2.3. Procedure

At T1, children completed the test of ANS, and mothers finished the home numeracy activities questionnaire. At T2, children were assessed on the mapping ability task, TEMA-3, and the intelligence. A group of trained graduate students in psychology in China conducted the administration of the measures. Extensive explanations were provided to the participants if they had questions during data collection. Children tests were conducted in a random order, in a quiet room in the kindergarten. All procedures performed in the study were approved by the University's Research Ethics Committee. Consent was obtained from all participants and their mothers through the kindergarten. After completing all the tests at Time 2, all participated children received a picture book for their

Table 1
Descriptive statistics and correlations among observed variables.

Variables	*M*	*SD*	Skewness	Kurtosis	1	2	3	4	5	6	7	8	9	10	11
1. Gender	0.46	0.50	0.18	−1.97	1										
2. Age	55.03	3.73	0.20	−1.34	0.02	1									
3. ANS$_1$	86.74	8.12	−1.10	0.80	0.19	0.17	1								
4. Number skills$_1$	1.97	0.74	−0.28	−0.48	−0.01	0.16	0.06	1							
5. Number books$_1$	1.54	0.90	0.25	−0.73	−0.18	0.20	0.10	0.71**	1						
6. Games$_1$	1.32	0.95	0.34	−1.03	−0.13	0.05	0.02	0.55**	0.54**	1					
7. Applications$_1$	1.28	0.97	0.67	−0.03	−0.08	0.02	0.11	0.48**	0.45**	0.51**	1				
8. Mapping ability$_2$	58.70	11.04	−0.03	0.50	−0.04	0.11	0.22*	0.27**	0.26*	0.30**	0.07	1			
9. Formal mathematics skills$_2$	11.22	5.45	0.84	−0.01	−0.30**	0.30**	0.29**	0.18	0.25*	0.23*	0.09	0.38**	1		
10. Informal mathematics skills$_2$	29.74	4.01	−0.55	0.60	−0.19	0.23*	0.31**	0.27*	0.31**	0.23*	0.08	0.32**	0.77**	1	
11. Intelligence$_2$	17.37	3.83	−0.30	2.41	−0.07	0.19	0.40**	0.12	0.14	0.11	−0.05	0.23*	0.48**	0.37**	1

Notes: Gender: 0 = boy, 1 = girl; ANS$_1$ = Approximate Number System at Time 1. *$p < .05$; ** $p < .01$.

participation.

3. Results

3.1. Preliminary analyses

Descriptive statistics and correlations among observed variables are presented in Table 1. Child gender (0 = boys, 1 = girls) was negatively correlated with formal mathematics skills, $r = -0.30$, $p < .01$, but not with informal mathematics skills, $r = -0.19$, $p > .05$, indicating that boys had higher formal mathematics skills than girls. Child age was positively correlated with formal mathematics skills, $r = 0.30$, $p < .01$, and informal mathematics skills, $r = 0.23$, $p < .05$. T2 intelligence was significantly related to T2 formal and informal mathematics skills, $r = 0.48$, $r = 0.37$, $ps < .01$, respectively. Child gender, age, and intelligence were statistically controlled in the following analyses.

ANS at T1 was significantly correlated with mapping ability, formal and informal mathematics skills at T2. In addition, T1 home numeracy activities with the exception of applications activities were generally associated with mapping ability, formal and informal mathematics skills at T2. Specifically, number books and games at T1 were significantly associated with mapping ability, formal and informal mathematics skills at T2; number skills at T1 were significantly associated with mapping ability and informal mathematics skills at T2. In addition, mapping ability at T1 was significantly associated with formal and informal mathematics skills at T2. These bivariate correlations raised the possibility of mediation paths via mapping ability leading to mathematics skills, which was tested in the next step of analysis.

3.2. Structural equation models

The analysis examined the direct and indirect effects (corresponding to Hypotheses 1 and 2, respectively) from ANS and home numeracy activities to children's formal and informal mathematics skills. Due to the unrelatedness of applications activities with mapping ability and two types of mathematics skills, we excluded applications activities and used number skills, number books and games as indicators of home numeracy activities when running the model (see Fig. 1). Bias-corrected bootstrap confidence intervals were computed to assess the significance of the indirect effects. If the 95 % confidence interval contain 0, the indirect effect was not considered significant (Preacher & Hayes, 2008). Children's gender was added as a predictor of formal mathematics skills, and age and intelligence were added as predictors of formal and informal mathematics skills. The pathway modes were run in Mplus 7.4.

The model achieved a good data-model fit, χ^2 (21) = 17.82, $p > .05$, RMSEA = 0.00, CFI = 1.00, TLI = 1.03, SRMR = 0.07. The results showed that, T1 ANS significantly predicted T2 informal mathematics skills, $\beta = 0.22$, $p < .05$, but not formal mathematics skills, $\beta = 0.15$, $p > .05$. T1 home numeracy activities significantly predicted T2 informal mathematics skills, $\beta = 0.21$, $p = .06$, but not formal mathematics skills, $\beta = 0.06$, $p > .05$. T1 ANS and T1 home numeracy activities both significantly predicted T2 mapping ability,

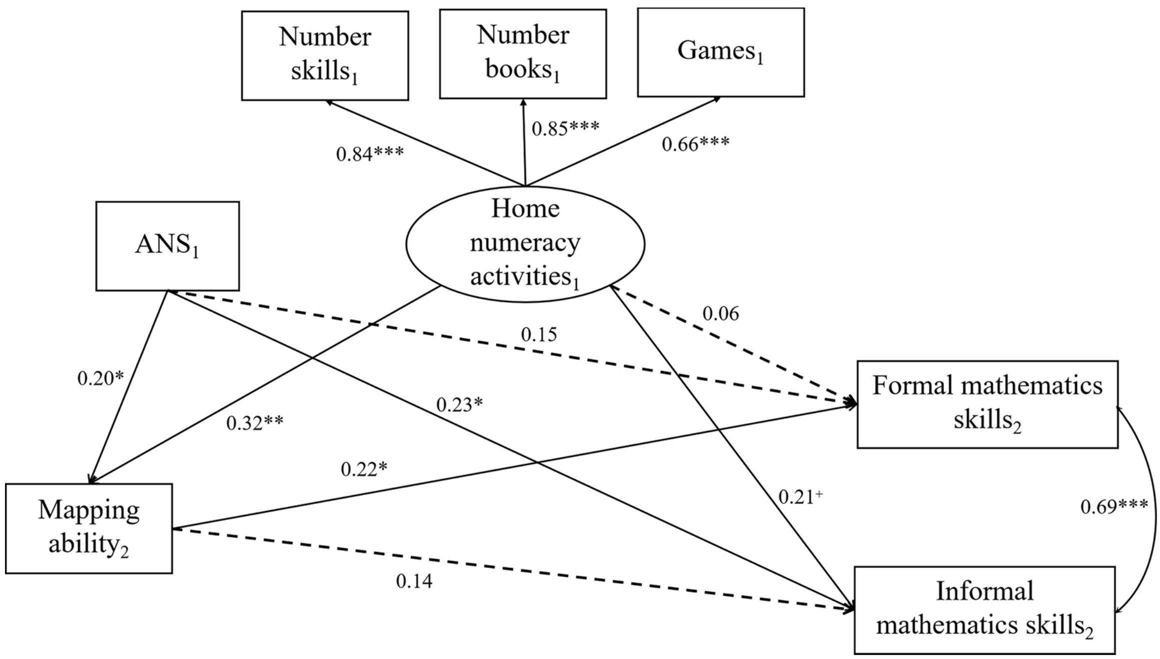

Fig. 1. T1 ANS and T1 home numeracy activities associated with T2 formal and informal mathematics skills through T2 mapping ability. + $.05 < p < .08$, * $p < .05$, ** $p < .01$, *** $p < .001$.

$\beta = 0.20$, $p < .05$, $\beta = 0.32$, $p < .01$, respectively. T2 mapping ability significantly predicted T2 formal mathematics skills, $\beta = 0.23$, $p < .05$, but not T2 informal mathematics skills, $\beta = 0.14$, $p > .05$. Thus, Hypothesis 1 is supported. In addition, the results showed that the indirect effect of T2 mapping ability on the association between T1 ANS and T2 formal mathematics skills was significant, $\beta = 0.05$, $SE = 0.03$, 95 % CI = [.004, .138]. The indirect effect of T2 mapping ability on the association between T1 home numeracy activities and T2 formal mathematics skills was also significant, $\beta = 0.07$, $SE = 0.04$, 95 % CI = [.014, .194]. T1 ANS and T1 home numeracy activities did not indirectly predict T2 informal mathematics skills by T2 mapping ability, $\beta = 0.03$, $SE = 0.03$, 95 % CI = [-.006, .096]; $\beta = 0.05$, $SE = 0.04$, 95 % CI = [-.015, .142]. Therefore, Hypothesis 2 is supported.

Taken together, these results indicated similar mediation patterns of mapping ability on the relations between ANS and mathematics skills, and between home numeracy activities and mathematics skills. Specifically, T1 ANS and home numeracy activities (i.e., number skills, number books, and games) directly predicted T2 informal mathematics skills, and indirectly predicted T2 formal mathematics skills via T2 mapping ability; and this intermediate path did not establish for T2 informal mathematics skills.

4. Discussion

The main aim of the present study was to examine the relationship between ANS, mapping ability, home numeracy activities, and mathematics skills in children, with a special focus on mapping ability as mediating the relationship between ANS and early mathematics skills as well as the relationship between home numeracy activities and mathematics skills. To gain a more nuanced picture of their associations, the present study distinguished between formal and informal types of early mathematics skills. The results showed that the predictive variables had differential effects on formal versus informal mathematics skills.

4.1. Correlations of ANS, mapping ability and home numeracy activities with mathematics skills

The current findings demonstrated that ANS, mapping ability, and home numeracy activities serve as the foundation of early mathematics skills. First, ANS at T1 was positively correlated with early mathematics skills at T2. These results were consistent with findings of previous meta-analyses (Chen & Li, 2014; Fazio et al., 2014; Schneider et al., 2017). Second, mapping ability at T1 was also found to be correlated with early mathematics skills at T2, which was in line with previous research (Jiménez Lira et al., 2017; Mundy & Gilmore, 2009). In addition, T1 home numeracy activities with the exception of applications activities were generally associated with formal and informal mathematics skills at T2. Specifically, number skills activities were correlated with informal mathematics skills. It is possible that these activities help children to learn different ways of representing quantities, such as by connecting quantities to size, length, weight, or number, which contribute to the understanding of non-symbolic quantities. Additionally, children may receive symbolic number knowledge in number skills activities, which can promote formal mathematics skills (Skwarchuk et al., 2014). However, the present study did not find the positive relationship between number skills activities and formal mathematics skills. Children might need to be exposed to higher frequency of number skills activities for a better mastery of formal mathematics knowledge. In addition, number books and games activities were found to be correlated with formal and informal mathematics skills. It is possible that these activities help to subtly promote children's understanding of the concept of numbers. In number books activities, parents may provide the role of scaffolding for children to understand the symbol and non-symbol meanings of numbers in the books (Mutaf Yildiz et al., 2018b). In games activities, such as board games, parents may guide children to transcode dots into numbers (Ramani & Siegler, 2011). These activities make children become more proficient in numbers. However, in this study, applications activities did not correlate with formal and informal mathematics skills. Compared to play activities, Chinese parents tend to use more directives and commands in assisting children during the preschool years (Huntsinger & Jose, 2009; Huntsinger et al., 1997). The low frequency of applications activities may be the reason for the unrelatedness with two types of mathematics skills.

4.2. The indirect effects of ANS and home numeracy activities on formal mathematics skills

The effects of ANS on formal mathematics skills were fully mediated by mapping ability. This finding solidified the previous empirical studies (Jang & Cho, 2018; Libertus et al., 2016; Rittle-Johnson et al., 2017; Wong et al., 2016) and the model of mathematical learning (Geary, 2013). Children are born with ANS, which including the successor principle, and facilitate generate cardinal values for magnitudes (vanMarle et al., 2018). When learning symbolic mathematics, they need to map number words or Arabic numerals to ANS to acquire quantitative meanings (Mundy & Gilmore, 2009; vanMarle et al., 2018).

In addition, this study found that mapping ability also mediated the relationship between home numeracy activities and children's formal mathematics skills. This result suggests that math learning experience is related to the accuracy of mapping ability (Castronovo & Göbel, 2012; Nys et al., 2013). There are many home numeracy activities that tap into different ways of representing numbers, such as board game with dice, in which children may transcode the dots into numbers (Ramani & Siegler, 2011; Susperreguy, Di Lonardo Burr et al., 2020). With exposure to these activities, children have more opportunity to facilitate connections between number symbols and their relevant magnitudes (Ramani & Scalise, 2020), which were associated with their mapping ability. Moreover, children with higher mapping ability are more proficient to get quantitative meanings and more likely to shift to a higher level of competence (Krajewski & Schneider, 2009), which might lead to better mathematical performance, thus yielding further evidence in favor of relations between mapping ability and formal mathematics skills (Jiménez Lira et al., 2017; Libertus et al., 2016).

4.3. The direct effects of ANS and home numeracy activities on informal mathematics skills

Unlike their indirect impact on formal mathematics skills, presumably mediated by mapping ability, ANS and home numeracy activities directly predicted informal mathematics skills. Informal mathematics skills involve knowledge of non-symbolic numerical magnitudes (Skwarchuk et al., 2014; Zhang et al., 2020). In comparison to formal mathematics skills, informal mathematics skills involve less mapping process (Libertus et al., 2016). It was easy for children to use ANS to complete informal mathematics tasks, such as non-symbolic magnitude comparison and non-symbolic arithmetic (vanMarle, 2015; vanMarle et al., 2018). Furthermore, children who were frequently engaged in home numeracy activities were more likely to master non-symbolic knowledge (Gunderson & Levine, 2011). For instance, in some of these activities such as counting, parents support children's learning with the help of concrete objects. Children enhance their understanding of non-symbolic numbers by directly perceiving and operating. Thus, the influences of ANS and home numeracy activities on children's informal mathematics skills were more direct.

4.4. Limitations and implications

Several limitations of present study are worth discussing. Researchers believed that the object tracking system (OTS) also contributes to the acquisition of number words meanings (vanMarle, 2015; vanMarle et al., 2018). OTS is a precise mechanism by which a small number of objects are represented as distinct individual (Feigenson et al., 2004), and it provides principles such as successor principle to make the cardinal values available in ANS (vanMarle et al., 2018). Future studies should test a model that integrates the role of ANS and OTS. In addition, we only included mothers in this study. Studies found that fathers' involvement but not mothers' in informal home numeracy activities predicted children's mathematics skills (Huang et al., 2017; Liu et al., 2019). Future studies should examine if mothers and fathers have different influences in this model. Finally, we only tested this model with children of preschool years. Thus, we do not know if these influences could exist after children enter primary school. Future longitudinal studies should keep track of this aspect of young children's mathematical development up to primary schools.

Despite the limitations, the present study significantly contributed to our understanding of how ANS, mapping ability, and home numeracy activities jointly affect formal and informal mathematics skills in Chinese preschool children. The results demonstrated that ANS and home numeracy activities indirectly predicted formal mathematics skills through mapping ability and directly predicted informal mathematics skills. These findings may have practical implications. Parents of children with low ANS levels do not worry too much, because home numeracy activities can also promote children's mathematics skills. We suggest that all parents, not merely those whose children have low ANS levels, should consciously increase parent-child numeracy activities, pay attention to the cultivation of children's ability to map Arabic numerals onto arrays of objects with numerical meaning, and guide children to understand number words through communication and inspiration.

5. Conclusion

Overall, findings of the present study demonstrated that ANS, mapping ability, and home numeracy activities together provide foundations for early mathematics leaning. Specifically, the current study found that, in preschool children, ANS and home numeracy activities directly predicted informal mathematics skills and indirectly predicted formal mathematics skills through mapping ability.

Funding

This work was supported by the Programs Foundation of Shanghai Municipal Commission of Health and Family Planning (Grants 201540114) and Key Specialist Projects of Shanghai Municipal Commission of Health and Family Planning (Grants ZK2015B01).

Declaration of Competing Interest

None.

Acknowledgments

The authors would like to thank the children and their parents for participating in this study as well as all the experimenters' help in collecting data.

Appendix A

Home numeracy activities	Reliability	Factor loading	*M*	*SD*	Skewness
Number skills	0.88				
Identify names of numbers		0.64	1.70	1.28	0.09
Playing with number fridge magnets		0.59	1.03	1.20	0.73

(*continued on next page*)

(*continued*)

Home numeracy activities	Reliability	Factor loading	*M*	*SD*	Skewness
Counting objects		0.65	2.54	1.04	−0.68
Sort things by size, colour, or shape		0.74	2.15	1.13	−0.45
Counted down		0.65	2.07	1.28	−0.10
Learning simple sums		0.64	2.42	1.19	−0.54
Printing numbers		0.59	2.63	1.08	−0.55
Singing number songs		0.57	1.45	1.20	0.52
Ordering things by size, length, weight, or number		0.65	1.42	1.03	0.48
Discriminating different sizes and amounts		0.67	1.70	1.01	0.19
Number books	0.72				
Connect-the-dot activities		0.69	1.53	1.20	0.34
Using number activity books		0.70	1.78	1.12	−0.01
Reading number storybooks		0.71	1.29	1.08	0.44
Games	0.60				
Playing board games with die or spinner		0.58	0.99	1.08	0.86
Playing computer games involving counting or arithmetic		0.75	1.38	1.20	0.33
Applications	0.58				
Talking about money when shopping		0.63	1.64	1.27	0.27
Using calendars and dates		0.66	0.83	1.06	1.09

References

Blevins-Knabe, B., Austin, A. B., Musun, L., Eddy, A., & Jones, R. M. (2000). Family home care providers' and parents' beliefs and practices concerning mathematics with young children. *Early Child Development and Care, 165*(1), 41–58. https://doi.org/10.1080/0300443001650104

Castronovo, J., & Göbel, S. M. (2012). Impact of high mathematics education on the number sense. *PloS One, 7*(4), e33832. https://doi.org/10.1371/journal.pone.0033832

Chen, Q., & Li, J. (2014). Association between individual differences in non-symbolic number acuity and math performance: A meta-analysis. *Acta Psychologica, 148*, 163–172. https://doi.org/10.1016/j.actpsy.2014.01.016

Fazio, L. K., Bailey, D. H., Thompson, C. A., & Siegler, R. S. (2014). Relations of different types of numerical magnitude representations to each other and to mathematics achievement. *Journal of Experimental Child Psychology, 123*, 53–72. https://doi.org/10.1016/j.jecp.2014.01.013

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*(7), 307–314. https://doi.org/10.1016/j.tics.2004.05.002

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition, 44*(1-2), 43–74. https://doi.org/10.1016/0010-0277(92)90050-r

Geary, D. C. (2013). Early foundations for mathematics learning and their relations to learning disabilities. *Current Directions in Psychological Science, 22*(1), 23–27. https://doi.org/10.1177/0963721412469398

Ginsburg, H. P., & Baroody, A. J. (2003). *TEMA: Test of early mathematics ability* (3rd ed.). Austin, TX: Pro-Ed.

Gunderson, E. A., & Levine, S. C. (2011). Some types of parent number talk count more than others: Relations between parents' input and children's cardinal-number knowledge. *Developmental Science, 14*(5), 1021–1032. https://doi.org/10.1111/j.1467-7687.2011.01050.x

Hu, L. T., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling A Multidisciplinary Journal, 6*, 1–55. https://doi.org/10.1080/10705519909540118

Huang, Q., Zhang, X., Liu, Y., Yang, W., & Song, Z. (2017). The contribution of parent-child numeracy activities to young Chinese children's mathematical ability. *The British Journal of Educational Psychology, 87*(3), 328–344. https://doi.org/10.1111/bjep.12152

Huntsinger, C. S., & Jose, P. E. (2009). Parental involvement in children's schooling: Different meanings in different cultures. *Early Childhood Research Quarterly, 24*(4), 398–410. https://doi.org/10.1016/j.ecresq.2009.07.006

Huntsinger, C. S., Jose, P. E., Liaw, F.-R., & Ching, W.-D. (1997). Cultural differences in early mathematics learning: A comparison of Euro-American, Chinese-American, and Taiwan-Chinese families. *International Journal of Behavioral Development, 21*(2), 371–388. https://doi.org/10.1080/016502597384929

Jang, S., & Cho, S. (2018). The mediating role of number-to-magnitude mapping precision in the relationship between approximate number sense and math achievement depends on the domain of mathematics and age. *Learning and Individual Differences, 64*, 113–124. https://doi.org/10.1016/j.lindif.2018.05.005

Jiménez Lira, C. J., Carver, M., Douglas, H., & LeFevre, J. A. (2017). The integration of symbolic and non-symbolic representations of exact quantity in preschool children. *Cognition, 166*, 382–397. https://doi.org/10.1016/j.cognition.2017.05.033

Kolkman, M. E., Kroesbergen, E. H., & Leseman, P. P. M. (2013). Early numerical development and the role of non-symbolic and symbolic skills. *Learning and Instruction, 25*, 95–103. https://doi.org/10.1016/j.learninstruc.2012.12.001

Krajewski, K., & Schneider, W. (2009). Early development of quantity to number-word linkage as a precursor of mathematical school achievement and mathematical difficulties: Findings from a four-year longitudinal study. *Learning and Instruction, 19*(6), 513–526. https://doi.org/10.1016/j.learninstruc.2008.10.002

LeFevre, J.-A., Skwarchuk, S.-L., Smith-Chant, B. L., Fast, L., Kamawar, D., & Bisanz, J. (2009). Home numeracy experiences and children's math performance in the early school years. *Canadian Journal of Behavioural Science, 41*(2), 55–66. https://doi.org/10.1037/a0014532

Li, X., McFadden, K., & DeBey, M. (2019). Is it DAP? American preschool teachers' views on the developmental appropriateness of a preschool math lesson from China. *Early Education and Development, 30*(6), 765–787. https://doi.org/10.1080/10409289.2019.1599094

Libertus, M. E., Feigenson, L., & Halberda, J. (2013). Numerical approximation abilities correlate with and predict informal but not formal mathematics abilities. *Journal of Experimental Child Psychology, 116*(4), 829–838. https://doi.org/10.1016/j.jecp.2013.08.003

Libertus, M. E., Odic, D., Feigenson, L., & Halberda, J. (2016). The precision of mapping between number words and the approximate number system predicts children's formal math abilities. *Journal of Experimental Child Psychology, 150*, 207–226. https://doi.org/10.1016/j.jecp.2016.06.003

Liu, Y., Zhang, X., Song, Z., & Yang, W. (2019). The unique role of father–child numeracy activities in number competence of very young Chinese children. *Infant and Child Development, 28*(4), e2135. https://doi.org/10.1002/icd.2135

Missall, K., Hojnoski, R. L., Caskie, G. I. L., & Repasky, P. (2015). Home numeracy environments of preschoolers: Examining relations among mathematical activities, parent mathematical beliefs, and early mathematical skills. *Early Education and Development, 26*(3), 356–376. https://doi.org/10.1080/10409289.2015.968243

Missall, K. N., Hojnoski, R. L., & Moreano, G. (2017). Parent–Child mathematical interactions: Examining self-report and direct observation. *Early Child Development and Care, 187*(12), 1896–1908. https://doi.org/10.1080/03004430.2016.1193731

Mundy, E., & Gilmore, C. K. (2009). Children's mapping between symbolic and nonsymbolic representations of number. *Journal of Experimental Child Psychology, 103*(4), 490–502. https://doi.org/10.1016/j.jecp.2009.02.003

Mutaf Yildiz, B. M., Sasanguie, D., Smedt, B. D., & Reynvoet, B. (2018a). Frequency of home numeracy activities is differentially related to basic number processing and calculation skills in kindergartners. *Frontiers in Psychology, 9*, 340. https://doi.org/10.3389/fpsyg.2018.00340

Mutaf Yildiz, B. M., Sasanguie, D., Smedt, B. D., & Reynvoet, B. (2018b). Investigating the relationship between two home numeracy measures: A questionnaire and observations during Lego building and book reading. *The British Journal of Developmental Psychology, 36*(2), 354–370. https://doi.org/10.1111/bjdp.12235
Naglieri, J. A., & Das, J. P. (2014). *CAS2: Cognitive assessment system*. Pro-Ed.
Nys, J., Ventura, P., Fernandes, T., Querido, L., Leybaert, J., & Content, A. (2013). Does math education modify the approximate number system? A comparison of schooled and unschooled adults. *Trends in Neuroscience and Education, 2*(1), 13–22. https://doi.org/10.1016/j.tine.2013.01.001
Peng, P., Yang, X., & Meng, X. (2017). The relation between approximate number system and early arithmetic: The mediation role of numerical knowledge. *Journal of Experimental Child Psychology, 157*, 111–124. https://doi.org/10.1016/j.jecp.2016.12.011
Preacher, K. J., & Hayes, A. F. (2008). Asymptotic and resampling strategies for assessing and comparing indirect effects in multiple mediator models. *Behavior Research Methods, 40*(3), 879–891. https://doi.org/10.3758/brm.40.3.879
Purpura, D. J., Baroody, A. J., & Lonigan, C. J. (2013). The transition from informal to formal mathematical knowledge: Mediation by numeral knowledge. *Journal of Educational Psychology, 105*(2), 453–464. https://doi.org/10.1037/a0031753
Ramani, G. B., & Scalise, N. R. (2020). It's more than just fun and games: Play-based mathematics activities for Head Start families. *Early Childhood Research Quarterly, 50*, 78–89. https://doi.org/10.1016/j.ecresq.2018.07.011
Ramani, G. B., & Siegler, R. S. (2011). Reducing the gap in numerical knowledge between low- and middle-income preschoolers. *Journal of Applied Developmental Psychology, 32*(3), 146–159. https://doi.org/10.1016/j.appdev.2011.02.005
Rittle-Johnson, B., Fyfe, E. R., Hofer, K. G., & Farran, D. C. (2017). Early math trajectories: Low-income children's mathematics knowledge from ages 4 to 11. *Child Development, 88*(5), 1727–1742. https://doi.org/10.1111/cdev.12662
Schneider, M., Beeres, K., Coban, L., Merz, S., Schmidt, S. S., Stricker, J., et al. (2017). Associations of non-symbolic and symbolic numerical magnitude processing with mathematical competence: A meta-analysis. *Developmental Science, 20*(3). https://doi.org/10.1111/desc.12372. e12372.
Shanghai Statistical Yearbook. (2019). *Basic statistics of whole city households in 2015 to 2018*. Retrieved June 11, 2021, from http://tjj.sh.gov.cn/tjnj/nj19.htm?d1=2019tjnj/C1001.htm.
Sherman, J., & Bisanz, J. (2009). Equivalence in symbolic and nonsymbolic contexts: Benefits of solving problems with manipulatives. *Journal of Educational Psychology, 101*(1), 88–100. https://doi.org/10.1037/a0013156
Skwarchuk, S. L., Sowinski, C., & LeFevre, J. A. (2014). Formal and informal home learning activities in relation to children's early numeracy and literacy skills: The development of a home numeracy model. *Journal of Experimental Child Psychology, 121*, 63–84. https://doi.org/10.1016/j.jecp.2013.11.006
Skwarchunk, S. L. (2009). How do parents support preschoolers' numeracy learning experiences at home? *Early Childhood Education Journal, 37*(3), 189–197. https://doi.org/10.1007/s10643-009-0340-1
Susperreguy, M. I., Di Lonardo Burr, S., Xu, C., Douglas, H., & LeFevre, J.-A. (2020). Children's home numeracy environment predicts growth of their early mathematical skills in kindergarten. *Child Development, 91*(5), 1663–1680. https://doi.org/10.1111/cdev.13353
Susperreguy, M. I., Douglas, H., Xu, C., Molina-Rojas, N., & LeFevre, J.-A. (2020). Expanding the Home Numeracy Model to Chilean children: Relations among parental expectations, attitudes, activities, and children's mathematical outcomes. *Early Childhood Research Quarterly, 50*, 16–28. https://doi.org/10.1016/j.ecresq.2018.06.010
Träff, U., Olsson, L., Skagerlund, K., & Östergren, R. (2020). Kindergarten domain-specific and domain-general cognitive precursors of hierarchical mathematical development: A longitudinal study. *Journal of Educational Psychology, 112*(1), 93–109. https://doi.org/10.1037/edu0000369
vanMarle, K. (2015). Foundations of the formal number concept: How preverbal mechanisms contribute to the development of cardinal knowledge. *Mathematical Cognition and Learning, 1*, 175–199. https://doi.org/10.1016/b978-0-12-420133-0.00007-7
vanMarle, K., Chu, F. W., Mou, Y., Seok, J. H., Rouder, J., & Geary, D. C. (2018). Attaching meaning to the number words: Contributions of the object tracking and approximate number systems. *Developmental Science, 21*(1), e12495. https://doi.org/10.1111/desc.12495
Wei, W., Li, Y., & Su, H.-Y. (2020). Predicting the growth patterns in early mathematics achievement from cognitive and environmental factors among Chinese kindergarten children. *Learning and Individual Differences, 79*, 101841. https://doi.org/10.1016/j.lindif.2020.101841
Wong, T. T.-Y., Ho, C. S.-H., & Tang, J. (2016). The relation between ANS and symbolic arithmetic skills: The mediating role of number-numerosity mappings. *Contemporary Educational Psychology, 46*, 208–217. https://doi.org/10.1016/j.cedpsych.2016.06.003
Zhang, X., Hu, B. Y., Zou, X., & Ren, L. (2020). Parent–child number application activities predict children's math trajectories from preschool to primary school. *Journal of Educational Psychology, 112*(8), 1521–1531. https://doi.org/10.1037/edu0000457
Zippert, E. L., Douglas, A.-A., Smith, M. R., & Rittle-Johnson, B. (2020). Preschoolers' broad mathematics experiences with parents during play. *Journal of Experimental Child Psychology, 192*, 104757. https://doi.org/10.1016/j.jecp.2019.104757