Encountering novel problems continuously, being encouraged to seek comprehension, freedom from urgent need for rewards, and dialogical interaction all aid mathematical understanding.

Social and Motivational Bases for Mathematical Understanding

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Distinguishing Between Adaptive and Routine Experts

The development of mathematical cognition is undoubtedly based on learners' experience, more specifically on practice in solving mathematical problems. It can be conceptualized as a process of acquiring expertise—that is, the accumulation and reorganization of domain-specific knowledge through problem solving. Recent studies on expert-novice differences in knowledge-rich domains such as physics and mathematics have shown that experts, using their rich and well-organized body of knowledge, generate an appropriate representation of a problem so that they can handle it easily to solve the problem (Chi, Glaser, and Rees, 1982; Glaser, 1986).

However, not all experts are flexible enough to be able to solve novel types of problems, even within the domain in which they have acquired expertise. While some learners are flexible in their use of those mathematical formulae and computation procedures they know, others can apply their problem-solving skills efficiently but only to the types of problems they have practiced routinely.

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Hatano and Inagaki (1986) attribute this difference in flexibility or adaptiveness to the extent of conceptual knowledge possessed. People who are regarded as experts in the target domain have a body of procedural knowledge needed to solve familiar types of problems promptly. They have procedures for making judgments as well as executing actions. However, they may not have conceptual knowledge, which is defined here as "more or less comprehensive knowledge of the nature of the object" of the procedures. The object may be a physical entity, for example, a device people operate many times, a plant they raise, and so on. The knowledge about the entity is often called a mental model (Gentner and Stevens, 1983)—a set of properties of the object, which people can use in mental simulations. The object may also be a cognitive entity characterized by its rich relationships (Hiebert and Lefevre, 1986), such as the decimal system of numbers.

Because of conceptual knowledge, which enables one to find the meaning of each step of the procedure in terms of the object's properties and their changes, one can understand how and why a given procedure works. More specifically, one can explain why the procedure is valid—that is, one can obtain what Greeno (1980) calls explicit understanding. With conceptual knowledge one can also judge not only the conventional version but also variations of the procedure as appropriate or inappropriate and then modify the procedure according to changes in constraints—that is, one can achieve implicit understanding (Greeno, 1980). For example, the conceptual knowledge of the decimal number system is the basis for explaining the borrowing procedure, differentiating "buggy algorithms" from unusual but valid procedures, and applying flexibly the procedure of multidigit subtraction.

In this way, we distinguish adaptive experts—those who have acquired rich conceptual knowledge—from routine experts—those who have not acquired such knowledge. While the latter are also experts by virtue of their speed and accuracy in solving routine problems, they are not able to "invent" new procedures. All they can do when given a novel type of problem or an apparently familiar one under modified conditions is to make minor adjustments, relying on trial and error.

Two comments are added to avoid possible misinterpretations: First, the skills of routine experts—applying procedures without conceptual knowledge—are not useless. To be a competent problem solver, one has to know how to apply the procedure and when to do so, but one does not have to go beyond this. We can solve a great number of mathematical problems using the right procedure at the right time without having the corresponding conceptual knowledge or understanding based on it. Very few of us in fact can explain why a given procedure (for instance, "To divide fractions, reverse the numerator and denominator of the divisor and multiply") works, though we believe it is valid and we can apply it efficiently. Lack of conceptual knowledge becomes a serious deficit only

when unusual, novel types of problems are posed. Thus, after having applied a procedure many times successfully, we may lose interest in knowing why the procedure works, because we take it for granted.

Second, conceptual knowledge may be incomplete or incorrect. A person may know that an orchid is similar to a cactus without being able to specify the differences between the two. Conceptual knowledge may involve false elements, as revealed most clearly in the case of so-called misconceptions, which contradict some known facts yet explain others. For example, many children believe that a potted plant can grow if it is given only water.

How and When Is Conceptual Knowledge Acquired?

No one has proposed a well-articulated theory, but typically conceptual knowledge is acquired by the process of constructing, elaborating, or revising the model so that it will plausibly explain a set of observed relationships. This process accompanies abduction, that is, the constrained generation of explanatory hypotheses based on limited data.

Consider an example of this process of abduction and of the construction of a model. After accumulated experience of growing flowers, a five-year-old girl stated, "Flowers are like people. If flowers eat nothing, they will fall down of hunger. If they eat too much, they will be taken ill" (Motoyoshi, 1979). The child generated explanations for her observations—that is, either giving no water or too much water makes flowers wither—by personifying flowers. In this process, she has developed a model of flowers as having a human-like structure. She chose this explanation from among a great many possible explanations, probably because her induction was constrained by the tendency to transfer knowledge about humans to all other living things, which is common among young children (Carey, 1985; Hatano and Inagaki, 1987a).

When is conceptual knowledge likely to be acquired? It is assumed that conceptual knowledge cannot be transmitted verbally or graphically. In other words, it must be constructed by each individual, though the process of construction can be guided to some extent by direct or indirect teaching. Its construction—formation, elaboration, or revision—is likely to occur, often as a by-product, when one seeks causal explanations for a given set of observed connections (for example, why a given procedure produces particular results). One is seldom engaged in activities aiming at the construction per se, though once constructed, conceptual knowledge plays an important role in solving various types of novel problems.

A Model of Adaptive Expertise

Under what circumstances does one construct rich conceptual knowledge and become an adaptive expert? What conditions tend to lead one to a routine expert, even after applying procedures many times to solve a large number of problems in a given domain?

How likely one is to acquire conceptual knowledge depends on the nature of the object and procedures for dealing with it. Among others, the following two cognitive conditions seem critical: (1) more or less appropriate models of the object can easily be obtained, and (2) steps of the procedures can easily be separated and manipulated. Since this acquisition is a process of building a model and checking it with the data, a good candidate model, even if very tentative and implicit, must be available. Knowledge may be obtained primarily through perception, as a somewhat vague image of the object (for example, we can learn how a water-drinking doll works by breaking it up and observing its parts closely), and by verbal and graphic description of the object's major characteristics (for example, a blueprint of a machine helps us understand it). Or knowledge may be derived indirectly on the basis of its functions or reactions. In the latter case, it is usually borrowed from elsewhere, as in the above example of personification of flowers. Knowledge acquisition also required multiple observations in which the procedure is broken down into steps or components and some are varied more or less systematically. In order to check a hypothetically assigned meaning, one has to change or omit the critical step or component. Such system observation, most evident among scientists, is needed to some extent in acquiring conceptual knowledge.

Although these two cognitive conditions are necessary, they are far from sufficient for the acquisition of conceptual knowledge and adaptive expertise. It is difficult to explain why very few students become adaptive experts in terms of cognitive conditions alone. In addition to cognitive conditions, motivational conditions seem critical. Assuming cognitive conditions are satisfied, one is likely to become an adaptive expert if and only if he or she is motivated to understand why procedures work while using them for problem solving.

When Are We Motivated to Understand? My colleague and I (Hatano and Inagaki, 1987b) have tried to formulate a process model of the arousal of motivation for comprehension drawing on Berlyne's theory of epistemic behavior (Berlyne, 1963, 1965a, 1965b). This theory assumes that, since human beings are intrinsically motivated to understand the world, "cognitive incongruity"—that is, a state where a person feels that his or her comprehension is intolerably inadequate—motivates a person to pursue subjectively adequate comprehension or satisfactory explanations. Cognitive incongruity induces enduring comprehension activity, including seeking further information from the outside, retrieving another piece of prior knowledge, generating new inferences, examining the compatibility of inferences more closely, and so on.

Three types of cognitive incongruity are distinguished. One is surprise, which is induced when a person encounters an event or information that disconfirms a prediction based on prior knowledge. A person will be motivated to understand why the prediction has failed and how to repair the prior knowledge by incorporating the new information. Another is perplexity, which is induced when a person is aware of equally plausible but competing ideas (predictions, assertions, explanations, and so forth) related to the target object or procedure. In this case one seeks further information in order not only to choose one of the alternatives but also to find justifications for the choice. The third is discoordination. This is the awareness of a lack of coordination among some or all of the pieces of knowledge involved. In other words, it is induced when one recognizes that though pieces of knowledge about the target are available, they are not well connected or that other pieces of related information cannot be generated by combining or in any way transforming the existing ones.

It should be noted, however, that the objective lack or inadequacy of comprehension does not always induce cognitive incongruity, nor does cognitive incongruity always induce comprehension activity. In order for cognitive incongruity to occur, people must themselves recognize the inadequacy of their comprehension. To do this, they must be able to monitor their own comprehension.

There exist two limiting conditions to be fulfilled before cognitive incongruity leads to enduring comprehension activity. One limiting condition is that people realize the importance and possibility of comprehension. Only when people have confidence in their ability to understand and when they experience cognitive incongruity about a target they value (because it is relevant to their lives) are they likely to engage in comprehension activity. Otherwise, they will be reluctant to engage in comprehension activity (which requires much mental effort), and they may suppress the motivation to comprehend.

The other limiting condition is the freedom from any urgent external need—for material reward, positive evaluation, or definitely correct answers. Studies on the so-called undermining effects of extrinsic rewards have shown that promised or given rewards deteriorate both the quality of performance in the task and intrinsic interest (Lepper, 1983; Lepper and Greene, 1978). This suggests, though indirectly, the possibility that extrinsic rewards inhibit motivation for comprehension. The expectation of rewards may change the goal of ongoing cognitive activity from comprehension to obtaining such rewards (Inagaki, 1980).

From the process model described so far, we can deduce the following three conditions under which a student is likely to be motivated to comprehend procedures used for problem solving and thus to become an adaptive expert in a domain:

1. *One encounters novel types of problems continuously in the course*

of acquiring expertise. Whereas familiar types of problems can easily be solved by applying the known procedure in an algorithmic way, thus avoiding cognitive incongruity, novel types of problems tend to produce perplexity and discoordination. Finding that a proposed solution is wrong tends to induce surprise.

2. *One is encouraged to seek comprehension.* Encouragement of comprehension leads an individual to form metacognitive beliefs emphasizing the significance and capability of comprehension (at least in the domain in which expertise is acquired); this makes comprehension activity likely to occur when cognitive incongruity is induced.

3. *One is free from urgent need to obtain external reinforcement when solving problems.* One can pursue comprehension only when the pressure to obtain rewards is not very strong, because engaging in comprehension activity is seldom the surest and shortest way to rewards. When solving a problem correctly is vitally important, one is likely to concentrate on it, suppressing cognitive incongruity.

The process model described above implies that the arousal of motivation for comprehension depends heavily on prior knowledge. In order for incongruity to be induced, one has to have relevant pieces of well-established knowledge. Surprise is felt only after a firm expectation is derived from prior knowledge. Perplexity is induced when more alternatives than one are judged plausible in the light of prior knowledge. Discoordination may occur only when a fair amount of relevant knowledge is available for further processing. Furthermore, as suggested by Markman (1981), people can promptly recognize the inadequacy of comprehension only in the domains where they have acquired rich and well-structured knowledge—that is, in their "domains of expertise." Likewise, individuals have their own "domains of interest" in which they believe they are able to comprehend and also in which the comprehension is valuable and independent of external rewards. People are willing to engage in prolonged comprehension activity in these domains.

However, outside their domains of expertise and interest, people are unlikely to recognize the inadequacy of their comprehension, unlikely to engage in comprehension activity even when incongruity is aroused, and, as a consequence, unlikely to acquire knowledge through comprehension. This vicious cycle of people as cognitive systems cannot be broken by introducing external reinforcement, because people are likely to be attracted to seek it, moving further away from comprehension.

Dialogical Interaction Induces Comprehension Activity. Those activities that can amplify motivation for comprehension outside the domains of expertise and interest are social-interactional ones in most cases. Dialogical interactions, such as discussion, controversy, and reciprocal teaching, in which knowledge or comprehension is to be shared often serve to enhance comprehension activity. Miyake (1986) poses a good example of how dialogical interaction motivates persons to engage in prolonged comprehension activity that would not be induced without a partner. When asked to find why a sewing machine can make stitches, Miyake found that pairs of subjects spent as long as sixty to ninety minutes trying to integrate different perspectives and knowledge bases through discussion. One of the pair claimed to understand the device before long, but criticism by the partner created once again the state of nonunderstanding (cognitive incongruity) that motivated the pursuit of deeper levels of understanding.

Why is dialogical interaction effective in inducing comprehension activity even among those students who lack rich and well-organized knowledge? Such interaction (1) tends to produce and amplify surprise, perplexity, and discoordination by helping people monitor their comprehension; and (2) relates the less familiar domain to one's domains of expertise and interest.

Surprise can be aroused by asking a person to make a prediction and then giving information that clearly disconfirms it. Surprise can be heightened when the prediction is given openly and unmistakably in dialogue. Perplexity is induced when one finds different ideas among fellow participants in dialogical interaction. The presence of others expressing different ideas is especially advantageous for amplifying perplexity, because one has to confront them. It is harder to maintain as plausible those ideas one merely reads or is exposed to passively.

A person may experience discoordination in the process of trying to explain why his or her views are reasonable when asked for clarification or when the views are directly challenged or disputed. Why is discoordination induced in such situations? Inagaki (1986) offers three reasons: First, one has to verbalize (make explicit what has been known only implicitly) in the process of trying to convince or teach others. This will lead one to examine one's own comprehension in detail and thus become aware of any thus far unnoticed inadequacies in the coordination among those pieces of knowledge. Second, since persuasion or teaching requires the orderly presentation of ideas, one has to organize better intra-individually what has been known. Third, for effective argumentation or teaching, one must incorporate opposing ideas—that is, coordinate different points of view inter-individually between proponents and opponents or between tutors and learners. Strong discoordination occurs only when one struggles to coordinate, since it is practically impossible to coordinate all the pieces of information available at any given moment.

Discussion, controversy, or teaching satisfies the first limiting condition as well—that is, it can help one realize the importance and possibility of comprehension. First, any of these elements invites a person to "commit" to some ideas by requiring the person to state the ideas to others, thereby placing the issue in question in the domains of interest.

Second, the social setting makes the enterprise of comprehension meaningful. Unless extrinsic motivation (such as winning the debate) is so strong that is supercedes motivation for comprehension. this social aspect will make comprehension activity enduring.

The above discussion strongly suggests that dialogical interaction enhances motivation for comprehension and thus the construction of conceptual knowledge. In fact, a number of investigators with differing theoretical orientations have found that peer discussion and decision making facilitate meaningful learning, understanding, and cognitive growth (Inagaki, 1986; Perret-Clermont, 1980; Smith, Johnson, and Johnson, 1981).

Therefore, it is reasonable to assume that frequent dialogical interaction tends to lead to adaptive expertise. In addition to the three motivational conditions mentioned above, the following fourth condition might be appended:

4. *The procedures are used often in dialogical interaction.* One is likely to seek justifications and explanations much more often in dialogical interaction than in solitary activity.

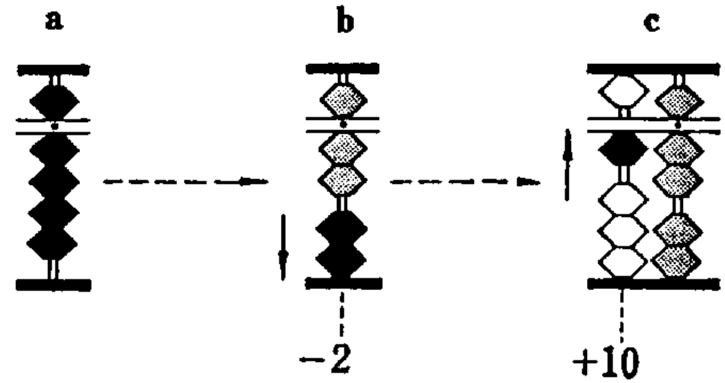
Applications of the Model

The above model of adaptive expertise can be applied to some domains of mathematical problem solving in an attempt to predict what type of experts students are likely to become from the four motivational conditions.

What Do Abacus Learners Fail to Acquire? If, in the course of acquiring expertise (1) one is given the same types of problems repeatedly, (2) efficiency is valued much but understanding is not, (3) always getting the right answer is required, and (4) procedures are seldom used in the dialogical context (that is, none of the four conditions for adaptive expertise is satisfied), one will necessarily become no more than a routine expert. A typical case of such routine expertise is expertise in mental abacus operation.

A large number of Japanese children learn abacus operation in addition to paper-and-pencil calculation. They use an abacus that has a five-unit bead in the upper section and four one-unit beads in the lower section in each column. The numbers 0 through 9 are represented by "entering" (pushing toward the dividing bar) different combinations of beads. Addition and subtraction are done by pushing beads toward and away from the bar, with a few rules regarding carrying and borrowing. For example, if an addend needs more beads than available, add 1 to one column left and subtract the complementary number-to-10 of the addend from the target column (see Figure 1).

Figure 1. An Example of Addition with an Abacus: 9 + 8 = 17



a. Enter 9.
b. You cannot add 8 in the ones column which has 9. So remove the complementary number-to-10 of 8 (that is 2) from the ones column.
c. Add 1 to the tens column.

Abacus operation, still used daily at small shops, is usually learned at a private school specialized for it, though it is sometimes acquired through informal observations and teaching. At the school students are given many problems for the four calculations without having the meaning of each step of calculation explained to them in detail. Since one can learn how to operate the instrument in just a few hours, training afterward is geared almost entirely to accelerating the speed of the operation.

Some abacus learners in fact become extraordinarily quick in calculation. For example, a fourth-grade girl my associate and I observed could solve 30 printed multidigit multiplication problems, 3 digits by 3 digits (for instance, 148 × 395) or 4 digits by 2 digits (3,519 × 42), in 58 seconds. This alone was surprising, but her net calculation time was even shorter—she needed the total amount of time for writing the answers down.

Several mechanisms are offered to explain this speed of calculation based on results from a large number of experiments (Hatano, forthcoming). First, a set of specific rules ("If addend 6 cannot enter, add 1 to one column left and subtract 4 from the target column") replaces the general rule that involves a variable ("the complementary number-to-10 of the addend" varies depending on the addend), and then a few such specific rules are merged into a single rule to get the final state directly ("If 7 is to be added to 6, add 1 to one column left and leave 3 at the target column"). Second, the application of these merged specific rules becomes more and more automatic.

Third, sensorimotor operation on physical representation of abacus beads comes to be interiorized as mental operation on a mental representation of an abacus. By this, the speed of the operation is no more limited by the speed of muscle movement. Fourth, a module-like system to represent mentally a number or series of digits in a form of the configuration of abacus beads, which is activated without any conscious effort or decision making, is established. Fifth, the mental imagery of abacus becomes simplified, eliminating properties unnecessary for calculation (such as the color of beads), so that the abacus can be manipulated even more quickly. Finally, monitoring of the operation is removed to use one's processing capacity to speed up the operation itself, since an expert's calculation is so fast that calculating twice is simpler than monitoring.

It should be noted that this process of acceleration of calculation speed results in a sacrifice of understanding and of the construction of conceptual knowledge. It is hard to unpack a merged specific rule to find the meaning of any given step (for example, leaving 3 at the target column when 7 is added to 6). A number is represented only in terms of a simplified image of beads that does not have rich meaning. No mental resources are used to reflect on why the procedure works, since this reflection would slow down calculation.

There have been a few studies suggesting that intermediate abacus learners do not know the meaning of the steps of abacus operation and that even experts lack conceptual knowledge. For example, interviews with abacus-learning third-graders about why certain steps were performed in the operation revealed that after a year of practice at an abacus school, they could explain the multidigit subtraction procedure no better than their agemates who just started the practice (Amaiwa and Hatano, 1983).

Amaiwa (1987) examined whether another group of third-graders who had practiced abacus operation for a year could repair their "buggy" paper-and-pencil calculation procedures by transferring the knowledge about the abacus procedure. She required the students alternately to solve the same problems with paper and pencil and with an abacus and found that many of them continuously made incorrect responses by the former procedure but correct responses by the latter. Amaiwa interpreted this to mean that since the students did not understand the meaning of steps of "base" abacus operation, they could not derive specific pieces of information to repair the "target" paper-and-pencil procedure.

In other unpublished studies with experts and junior experts, Amaiwa and I have found that they were not flexible in the use of their skills. When they were given multiplication and division problems, some of which could be solved by using simplifying strategies ($99 \times 38 \rightarrow 38 \times 100 - 38$; $9{,}250 \div 25 \rightarrow 9{,}250 \times 4 \div 100 \rightarrow 925 \times 4 \div 10$), they did not recognize this possibility and solved all the problems mechanically in

the same way, though their calculation was still much faster than that of ordinary college students. Another study revealed that the experts could not transfer their skills to nonconventional abacuses that were for a base 6 or 12 system. They performed no better with these abacuses than college students who had had negligible experience with the standard abacus. We must conclude that abacus operators apply the calculation procedures thousands of times without comprehending why the procedures work, probably because they have not constructed conceptual knowledge of the base 10 and other systems of numbers.

Abacus Operation and Street Mathematics. Abacus operation can be compared with other informal mathematical practices that have developed under different motivational conditions. It is interesting to do such a comparison because it helps clarify the significance of the four motivational conditions for determining the course of expertise. "Street mathematics" in Brazil will be used as an example (Carraher, Carraher, and Schliemann, 1985).

Apparently, abacus operation and street mathematics have much in common: (1) Both are used almost exclusively for commercial activities; (2) both can be acquired without systematic teaching; (3) both are outside of the "official knowledge" taught in school. However, they are radically different in "semantic transparency"—that is, in the clarity of the meaning of each calculation step. Steps of street mathematics, or "oral mathematics" (Carraher, Carraher, and Schliemann, 1987), in general, are clear in meaning, because the representations manipulated therein are information rich, and the ways of manipulation are analogous to actual activity dealing with goods, coins, and notes. For example, in order to find the price for twelve lemons of Cr$5.00 each, a nine-year-old child who was an expert street mathematician counted up by 10 (10, 20, 30, 40, 50, 60) while separating out two lemons at a time (Carraher, Carraher, and Schliemann, 1985). Quite to the contrary, representations of numbers on an abacus, though visibly concrete, are impoverished in meaning, and the way of manipulation is just mechanical.

Moreover, Brazilian children can flexibly use street mathematics or oral mathematics procedures. Oral computation procedures, often relying on decomposition and regrouping, generally reveal "solid understanding of the decimal system" (Carraher, Carraher, and Schliemann, 1987, p. 83)—that is, conceptual knowledge. Thus street mathematicians can be adaptive experts, while abacus operators are always routine experts.

These differences in knowledge come from differences in the cultural context of the practice and resultant motivational conditions for acquiring expertise. More specifically, what is different is the function of each mathematical practice in commercial activities. Street mathematics is basically a means by which a vendor and a customer reach an agreement as to the total price. It is an interpersonal enterprise that requires semantic transparency—otherwise the customer may be suspicious. Calculations cannot be performed very quickly, because they manipulate meaning-rich representations. However, the economy in which a young Brazilian vendor lives does not require high efficiency in calculation.

From the above analyses it can be assumed that (1) street mathematicians are posed novel types of problems fairly often because of changes in products, prices (as inflation increases), and customers' needs; (2) they are encouraged to seek comprehension as far as needed to explain to the customer the process of calculation; (3) accuracy of calculation is required, but not excessively, because its semantic transparency helps the vendor and customer recognize possible errors in calculation; and (4) calculation is done mostly in dialogical context. If these assumptions are correct, the motivational conditions for Brazilian vendors are radically different from those for abacus learners.

In contrast, Japanese abacus operation is basically a solitary activity in which operators handle large numbers quickly and accurately. Experienced abacus operators must be able to handle simplified representations, because the economy in which abacus operation developed required efficiency. A person or culture that values excessive efficiency must be content with simplified representations, giving up semantic transparency, understanding, and the construction of conceptual knowledge. Its operators are not interested in the semantic transparency of the calculation process either, because they believe that their skills ensure the correctness of the answer. Even when abacus operation is used in interpersonal situations of buying and selling, both the vendor and the customer are willing, in most cases, to accept the answers. Many Japanese customers and vendors seem to think that abacus operation is more dependable than calculation with a calculator.

Some Instructional Implications. From the preceding discussion of the nature of abacus operation as a form of nonschool, or "informal," mathematics, two instructional implications can be derived. First, teachers must keep in mind that not every mathematics procedure that emerges in nonschool settings can serve as a basis for understanding how and why the corresponding school mathematics procedure works. Carraher and colleagues (1985) maintain mathematics learning in daily life produces effective and meaningful procedures that can complement potentially richer and more powerful mathematical tools acquired in school at the expense of meaning. However, daily life procedures are in fact semantically transparent—that is, the meaning of each step is understood by students only when the motivational conditions for their acquisition enhance the construction of conceptual knowledge.

I doubt that all (or nearly all) daily routines are meaningful—that is, clear—regarding why each step is needed. In principle, "our lives are filled with procedures we carry out simply to get things done" (Hatano

and Inagaki, 1986, p. 266). Adults as well as children most likely perform some everyday problem-solving procedures only because they "work," without understanding the meaning of each step. If we repeat these steps hundreds of times, we can become quite skillful at them—that is, we can become routine experts. Pressing a key of a calculator to find the square root of a given number, like subtracting using an abacus, can be considered as one of such procedures.

Therefore, I doubt that it is always possible to find a semantically transparent informal procedure as the point of departure when we are to teach a formal one. We may need another strategy to make formal mathematics procedures meaningful. The second implication is relevant at this point. If we want to enable students to understand how and why school procedures work, we have to approximate the process of learning to the acquisition of street mathematics, not to expertise in abacus operation. In other words, we might encourage students to construct conceptual knowledge by providing the four motivational conditions that enhance it.

Although some traditional curriculum goals, such as efficiency in problem solving, accuracy, speed of calculation, and so on, must be sacrificed to some extent in order to pursue adaptive expertise, a majority of mathematics educators may be willing to do so if a model system of instruction for adaptive expertise is available. Such a model system is a Japanese science education method called Hypothesis-Experiment-Instruction, originally devised by Itakura (1962). A few people in Itakura's research group have applied the same instructional procedure to mathematics and limited areas of social studies. Hypothesis-Experiment-Instruction creates conditions for conceptual knowledge acquisition by maximally utilizing classroom discussion as well as by carefully sequencing problems.

The instructional procedure is as follows:

1. Students are presented with a question with three or four answer alternatives.
2. Students are asked to choose one answer by themselves.
3. Students' responses, counted by a show of hands, are tabulated on the blackboard.
4. Students are encouraged to explain and discuss their choices with one another.
5. Students are asked to choose an alternative once again (they may change their choices).
6. Students are allowed to test their predictions by observing an experiment (or reading a given passage).

Each answer alternative of a question represents a plausible idea, for example, a common misconception held by students as well as the correct one. Such a question will surely induce perplexity and discoordination.

It is also emphasized that students can clearly confirm or disconfirm their predictions by external feedback. Since questions arranged at the beginning part of a topic are likely to have right answers that contradict students' "modal" predictions based on their prior knowledge, they will experience surprise with the feedback.

If you visit a classroom in which Hypothesis-Experiment-Instruction is implemented successfully, you will be impressed by lively discussions in a large group of forty to forty-five students. You will recognize that the teacher is a facilitator who tries to stay as neutral as possible during students' discussion. This neutral attitude of the teacher is effective for encouraging students to seek comprehension and also for reducing their need to get external reinforcement, because pupils are invited to offer persuasive arguments to other pupils instead of seeking the right answer authorized by the teacher.

A few studies examining the effectiveness of this method (Hatano and Inagaki, 1987b; Inagaki and Hatano, 1968, 1977) have shown that the preceding six-step procedure tends to produce (1) higher student interest in testing their predictions or finding explanations, (2) a larger number of adequate explanations of the observed fact or stated rule, and (3) more prompt and more proper application of the learned procedure to a variety of situations. Many anecdotal reports strongly suggest that students taught in this method of instruction gradually come to think that understanding the how and why is more important than making the correct predictions. Therefore, although there is no direct evidence that students taught by Hypothesis-Experiment-Instruction tend to become adaptive experts in school science and mathematics, it seems a promising model system.

Conclusion

A model of adaptive expertise suggests four conditions under which students, while using procedures for solving a large number of problems, are motivated to comprehend the procedures and thus acquire conceptual knowledge. These conditions are (1) encountering novel types of problems continuously, (2) being encouraged to seek comprehension over efficiency, (3) freedom from urgent need to get external reinforcement, and (4) dialogical interaction. When none of these conditions are met, as in the case of abacus operation, students are very unlikely to acquire the conceptual knowledge enabling them to understand the meaning of procedures though they are skilled in the procedures. On the contrary, when these conditions are satisfied more or less adequately, as in the case of Brazilian street mathematics, learners are likely to achieve understanding and flexibility of procedures.

Although we need more direct and controlled tests, these motivational

conditions seem important for the development of mathematical understanding in instruction.

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