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# Mathematical precocity in young children: a neo-Piagetian perspective

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Two studies were conducted to explore mathematical precocity in young children. Study 1 examined mathematically gifted first and third graders' working memory development. The results showed that mathematically gifted children's working memory growth was similar to that expected of their age peers. Study 2 examined changes in mathematically gifted children's conceptual structures. Mathematically gifted children were roughly a year ahead of their age peers in the rate of development of conceptual structure in the numerical domain. A neo-Piagetian theory of intellectual development was used to explain these seemingly conflicting findings. The relation between working memory growth and conceptual development was discussed throughout the paper.

## Introduction

Despite an increasing body of literature on differences in intellectual ability, little is known about the nature and characteristics of mathematical precocity in young children. Among many questions of interest is whether gifted children's thinking is similar to that of their chronological age peers or mental age peers. Some argue that gifted children are born with atypical brain organization (O'Boyle *et al.*, 1991, 1994). This view suggests that gifted children begin their life's journey with intellectual advantages leading them to take unique developmental paths. Some others argue that the amount of deliberate practice is the determining factor of expertise (Ericsson, 2003; Ericsson *et al.*, 2005). This view claims little or no initial developmental advantage for gifted children.

A neo-Piagetian theory of intellectual development postulated by Case and his colleagues (Case, 1992; Case & Okamoto, 1996) views the intellectual development of gifted children as not radically different from that of their chronological age peers. The theory predicts a relatively universal pattern of development with an age-typical upper bound at each stage of development, with gifted children no exception. This is

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not to discredit gifted children's superior performance. Rather, gifted children are seen as possessing special ability, for example, for rapid learning of academic material or learning a particular type of content. The claim is that gifted children's advantages are domain- or task-specific with system-wide constraints setting an age-related upper bound.

According to Case's (1985; Case & Okamoto, 1996) theory, available working memory capacity is the primary limiter on children's developmental progression. Using this theory as a framework to view giftedness, it is reasonable to expect that the development of gifted children's working memory is similar to that of their age peers. Empirical evidence, however, is somewhat mixed. On a variety of working memory measures, some studies have found gifted children to perform in a manner consistent with their age peers (Globerson, 1985; Porath, 1992, 1996, 1997). Others reported superior performance of gifted children on various working memory measures (Schofield & Ashman, 1987; Segalowitz *et al.*, 1992; Saccuzzo *et al.*, 1994). Explanations for these inconsistent findings may involve the selection of working memory measures in relation to target cognitive abilities. The question remains open as to the role of working memory in the development of mathematical precocity in young children. This is the question addressed in the first study.

Whether or not gifted children are similar to their age peers in the rate of development of working memory, gifted children, by definition, do show superior performance in the area of their strengths. For mathematically precocious children, it is their number sense, speed and accuracy of solving numerical and quantitative problems, and their interest in these tasks that distance them from their age peers. All these factors seem to point to advanced conceptual understanding in the quantitative domain. That is, mathematical precocity may not advance general intellectual development but instead be related to children's increased schematic repertoire in the quantitative domain.

Mathematically gifted children's structural changes have not been examined empirically. One study that examined cognitive abilities of mathematically precocious children in kindergarten and first grade found their quantitative ability to be highly correlated with spatial ability but to a lesser degree with verbal ability (Robinson *et al.*, 1996). Coupled with findings from verbally precocious toddlers (Dale *et al.*, 1995), these findings suggest that young children are beginning to develop differentiated abilities early on—at least in the quantitative and verbal domains. Quantitative and spatial abilities, on the other hand, appear to co-exist in mathematically precocious children of this age.

The neo-Piagetian theory of intellectual development adopted for the current study may be able to shed light on these seemingly conflicting findings. Case postulated that intellectual development has its origin in the evolutionary history of the human organism, including the modular structure of the cortex. Human infants, during the first few months of life, parse their experience into a set of basic categories, which later become well-distinguished domains of knowledge. In the numerical domain, children by 4-years-old typically assemble a knowledge network consisting of counting principles as well as a network for making quantitative comparisons. The latter network relies heavily on children's ability to make visual–spatial judgments. By 6-years-old, children integrate these two knowledge networks to assemble a higher order structure, called a 'central numerical structure'.[1] This structure no longer requires reliance on spatial ability. Based on the foregoing analysis, it is reasonable to expect an overlap between quantitative and spatial abilities prior to 6-years-old. As children mature, these two abilities are expected to become differentiated. Young children's abilities in the quantitative and verbal domains, however, have little overlap in content. Therefore, it is reasonable to expect low correlations between them even among toddlers.

The second study reported in this article examined changes in mathematically gifted children's conceptual structures. The primary aim was to characterize the developmental changes that take place as preschoolers move from developing two independent schemas for counting (numerical) and quantity judgments (visual–spatial) to a unified conceptual structure that is unique to the numerical domain. Analyses therefore focused on children 5 to 7 years old who are expected by theory to be in the process of constructing a central numerical structure.

In summary, two aspects of mathematical precocity in young children were examined in this paper. The first was to find out if mathematically precocious children would show similar developmental patterns to their age peers when their working memory was assessed. We predicted that gifted children's working memory has age-related limits similar to their age peers. Gifted children have been known to develop efficient problem solving strategies and do so even within a short learning situation such as during testing. When constraints were placed so as to prevent gifted children from developing efficient strategies, we expected that their performance on working memory measures would vary little from that of their age peers. The second study addressed the question of how structural changes in the domain of numbers would take place in mathematically gifted young children. We predicted that mathematically gifted children would show an earlier integration in the domain in which they excel in comparison to their age peers. In doing so, we hoped to shed light on the close association found between quantitative and spatial abilities. Taken together, both these studies should help increase our understanding of mathematical precocity in young children.

## Study 1

*Mathematically gifted children's working memory*

Gifted children are known for their strong memory (Feldman, 1986; Gaultney *et al.*, 1996; Coyle *et al.*, 1998; Gaultney, 1998; Stumpf & Eliot, 1999). On working memory measures, however, studies have reported inconsistent findings. Some studies found gifted children to perform in a manner consistent with their age peers rather than their intelligence (Globerson, 1985; Porath, 1992, 1996, 1997). Others reported superior performance of gifted children on various working memory measures (Schofield & Ashman, 1987; Segalowitz *et al.*, 1992; Saccuzzo *et al.*, 1994).

Reasons for inconsistent findings include the complex interplay between particular cognitive advantages of gifted children and the type of working memory measure selected. When generally gifted children (e.g., high IQ scores) were administered working memory measures of digit span tasks, they tended to perform better than did their age peers. For example, Schofield and Ashman (1987) found that gifted children in fifth and sixth grades outperformed their age peers on the working memory measures of forward and backward digit span. Segalowitz *et al.* (1992) also reported that gifted seventh graders outperformed their age peers on the forward and backward digit span tasks. Both these studies selected gifted children on the basis of the WISC-R (Wechsler, 1991). The gifted fifth and sixth graders in the former study scored 125 or above on the vocabulary, object assembly and similarities subscales of the WISC-R; the gifted seventh graders in the latter study scored 135 or above on the full version of the WISC-R when they were in fourth grade. These studies therefore examined the relation between general IQ and digit-span working memory. The question of domain-specific advantage and working memory was not addressed. In addition, the digit span tasks from the WAIS-R (Wechsler, 1955) that required 'the subject to repeat back to the tester a series of digits read out *slowly* in the order presented' (Segalowitz *et al.*, 1992, p. 285; our italics) may have allowed gifted children to develop efficient strategies quickly, leading to superior performance. Gifted children's advantage when they are allowed to develop strategies has been noted (Globerson, 1985). This is also confirmed in Saccuzzo *et al.*'s (1994) study. They found that gifted children in second, third, fifth and sixth grades outperformed their age peers only when the task allowed sufficient time to develop strategies.

Gifted children appear to use what working memory capacity they have to good advantage, possibly by chunking certain concepts or using them in a flexible fashion. Domain-specificity may also be relevant. Robinson *et al.* (1996) found three measures of working memory—counting span, visual–spatial and verbal—to load on quantitative, spatial and verbal factors, respectively. The links between particular working memory measures and intelligence factors appear to be highly specific (Süss *et al.*, 2002). The studies that found gifted children to show similar levels of working memory to their age peers used domain-specific working memory measures (Porath, 1992, 1996, 1997). In one study, Porath (1992) examined 6-year-olds' performance on various measures of conceptual understanding and working memory in specific domains. She found that children who were generally, verbally and spatially gifted outperformed their chronological and mental age peers on measures of conceptual understandings in the logical reasoning, narrative and spatial domains, respectively. However, their performance on two working memory measures—counting span (Case, 1985) and visual–spatial span (Crammond, 1992) tasks—was not significantly different from that of their chronological age peers. These working memory measures were not only domain-specific but also designed to minimize opportunities to use knowledge and strategy.[2]

Building on the work of Porath (1992, 1996, 1997), the present study was designed to examine working memory of mathematically gifted children. We defined working memory as the amount of information one can retain while processing additional information. We used domain-specific measures of working memory that satisfied this definition. These measures were designed to prevent gifted children from developing efficient strategies. We therefore predicted that their performance on working memory measures would vary little from that of their age peers.

## *Method*

*Participants.* A total of 25 children participated in this study. There were 12 first graders (five boys and seven girls) and 13 third graders (eight boys and five girls). Their mean ages were 7.0 ($SD$=.33) and 8.9 ($SD$=.42) years for first and third graders, respectively. These students were recruited from a private elementary school located in an upper-middle class suburb of the greater Los Angeles area. The school is known for its academic rigor. In the area of mathematics, each of the entire classes of first and third graders in 2005 achieved a mean percentile rank score of 87 on the Stanford Achievement Test (SAT). The criterion for participation in the study was either superior mathematics performance on the SAT (i.e., 95 percentile or higher) or teacher recommendation. The first and third graders' mean SAT percentile scores in mathematics were 93 and 95, respectively.

*Measures and procedures.* Two measures of working memory as well as a measure of conceptual understanding of numbers were administered to all children. All measures were administered individually in a quiet room near children's classrooms.

*Working memory measure in the numerical domain.* The counting span task (Case, 1985; McKeough, 1992) was used as a measure of working memory in the numerical domain. Children were presented with sets of cards. Each card displayed a number of yellow and green dots placed in a random configuration. The task was to count only the green dots and report the cardinal value. Children were required to touch each green dot as they counted aloud so as to prevent them from rehearsal. As soon as they finished counting green dots, the target card was covered and children were asked to report the number of green dots they had just counted. At the easiest level (Level 1, typically passed by 4-year-olds), children were shown one card. As difficulty increased, the number of cards shown increased. That is, at the next level (Level 2, typically passed by 6-year-olds), children were shown two cards in a row and asked to remember the number of green dots on each of the two cards. At the next level (Level 3, typically passed by 8-year-olds), they were shown three cards at a time. The highest level was Level 6 with a set of six cards shown to children. If the child missed one number out of the sequence, the response was considered incorrect. Each level consisted of three trials. Testing continued until the child failed to respond correctly to all three trials. As for scoring, each child received a raw score (maximum possible was 18) as well as a level score (to qualify as passing a level, the child must respond correctly to two of the three trials at any level).

*Spatial working memory measure.* Crammond's (1992) visual–spatial span task was used as a measure of working memory in the spatial domain. Children were shown an 8 cm by 8 cm matrix (2 cm × 2 cm cells), with some cells colored black. Children were asked to touch each colored cell and remember its location. They were then asked to reproduce the locations of the colored cells on a blank matrix after the target matrix was hidden. At the easiest level (Level 1, 4-year-old level), children were shown a grid with only one cell colored. At the next level (Level 2, 6-year-old level), two cells were colored. As difficulty increased, the number of colored cells increased. The highest level was Level 5 where five cells were colored black. As in the counting span task, the child had to reproduce all of the locations correctly for each trial. Again, each level included three trials. Testing continued until the child failed to respond correctly to all three trials. Each child received a raw score (maximum possible was 15) as well as a level score (two correct of the three trials).

*Number knowledge task*. This task, adapted from Case and his colleagues' work (Case & Griffin, 1989; Case & Okamoto, 1996), was designed to assess children's understanding of the whole number system.[3] Items at the easiest level (Level 1, 4-year-old level) assessed children's understanding of counting principles (counting schema) as well as quantity comparisons (quantity judgment schema). Items at the next level (Level 2, 6-year-old level) assessed children's understanding of single-digit numbers. For example, children should be able to judge a quantitative magnitude based not on visual inspection but on numerical magnitudes (e.g., which is smaller, 8 or 6?). Items at the next level (Level 3, 8-year-old level) assessed children's understanding of two-digit numbers (e.g., which number is closer to 21: 25 or 18?). Items at the next level (Level 4, 10-year-old level) assessed children's integral understanding of whole numbers and operations (e.g., which difference is bigger; the difference between nine and six or the difference between eight and three?). Level 5[4] was the highest level, which included mental computations of multiplication, division and negative numbers such as, 'Which is closer to 1: −1.4 or 3.7?' The number of items at each of the five levels was five, nine, nine, nine and six, respectively. Testing continued until the child failed to respond correctly to more than half of the items at a level. Each child received a raw score (maximum possible was 38) as well as a level score (majority correct at each level).

### *Results*

Descriptive statistics by grade levels are presented in Table 1, including means (standard deviations) for the two working memory measures, the number knowledge task, and the Stanford Achievement Test (SAT), as well as the number of participants and their mean ages. It should be noted that the mean scores for the SAT did not reach the 95th + percentile rank. The criterion for participation was 95th+ percentile on the SAT or teacher recommendation. Although 60% of the participants were at the 95th percentile or higher on the SAT, the rest did not reach this criterion. The number knowledge task was thus administered in order to ensure that all of our participants would show advanced levels of numerical understanding. As shown in Table 1 as well as Figure 1, the first and third graders obtained mean

Table 1. Sample sizes, mean ages, and means (standard deviations) of gifted children by grade levels

| | | | | | Working memory | | | |
|---|---|---|---|---|---|---|---|---|
| | | | Number knowledge | | Counting span | | Visual–spatial | |
| Age group | Age in months | SAT | Raw score | Level score | Raw score | Level score | Raw score | Level score |
| First graders | 83.75 | 93.42 | 21.92 | 3.00 | 8.08 | 2.50 | 7.42 | 2.17 |
| ($n$=12) | (4.22) | (6.08) | (2.88) | (.43) | (2.07) | (.67) | (2.61) | (.58) |
| Third graders | 107.23 | 95.31 | 30.54 | 4.23 | 9.31 | 2.92 | 12.15 | 3.77 |
| ($n$=13) | (5.54) | (3.99) | (3.62) | (.73) | (2.02) | (.76) | (2.79) | (1.09) |

level scores of 3.00 and 4.23, respectively. According to the analysis by Case and colleagues (see Case & Okamoto, 1996), age-appropriate levels of performance on this task would be 2.5 and 3.5 for children in the first and third grades, respectively. That is, they showed levels of conceptual understanding roughly one year (for the first graders) and two years (for the third graders) ahead of their respective age peers. The gifted children in this study obtained significantly higher mean scores than expected for age (binomial tests; both $p<.01$, two-tailed).

The mean level scores derived from the two working memory measures were also designed to provide age-related indices. As in the number knowledge task, scores of 2.5 and 3.5 were the mean level scores that non-gifted children of comparable age should obtain. The mean level scores on the visual–spatial span task were 2.17 and 3.77 for the first and third graders, respectively. According to binomial tests, these means were not significantly different from those expected of their age peers. As for the counting span task, gifted first and third graders obtained mean scores of 2.5 and 2.92, respectively. The first graders' mean was at the level expected for their age whereas the third graders' mean was below the expected age level ($p<.05$, binomial test, two-tailed).

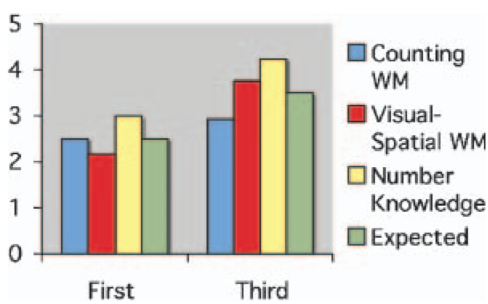


Figure 1. Mathematically gifted children's performance on working memory and number knowledge measures

To better understand the relations between mathematical giftedness and working memory, zero order and age-partialled correlation coefficients were computed (Table 2). The two measures of working memory showed a low correlation of .195 (partial $r$=.072) to each other. The number knowledge task was significantly correlated with each of the working memory measures: $r$=.475 ($p$<.05) for the counting span task and $r$=.683 ($p$<.01) for the visual–spatial task. These relations held up even when age was partialled out. Finally, the Stanford Achievement Test (SAT) and the number knowledge task showed low correlations. The SAT was not correlated with either of the working memory measures.

### *Discussion*

This study examined mathematically gifted children's working memory. The question of interest was to find out whether or not mathematically gifted children would show similar levels of working memory development to their age peers. The study participants—mathematically gifted first and third graders—were tested on two measures of working memory and a measure of conceptual understanding of number. The results for gifted children's conceptual understanding of number confirmed that our participants in fact possessed advanced levels of understanding in the domain of number. The obtained scores of 3.0 and 4.23 for first and third graders, respectively, were statistically higher than the expected values for average children. These scores roughly correspond to one year ahead of age expectation for the first graders and nearly two years for the third graders. In contrast, the findings for the participants' working memory did not show advantages for gifted children. These findings are consistent with earlier findings by Porath (1992, 1996, 1997) who found children gifted in the narrative and spatial-artistic domains to show similar levels of working memory to their age peers. These findings are in stark contrast to the advanced levels of working memory for gifted children reported by Schofield and Ashman (1987) and Segalowitz *et al.* (1992). One reason that may account for the conflicting results is whether or not the selected tasks allow children to develop efficient strategies during testing. The current study and those by Porath used the tasks that were designed to minimize opportunity to develop strategies whereas others did not. Segalowitz *et al.*'s study, for example, stated that the interviewer intentionally read a series of digits slowly, allowing gifted children to

Table 2. Zero-order correlations (below diagonal) and age-partialled correlations (above diagonal) among measures

| Measure | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1. Counting span working memory | — | .072 | .452* | .120 |
| 2. Visual–spatial working memory | .195 | — | .444* | .181 |
| 3. Number knowledge | .475* | .683** | — | −.021 |
| 4. SAT | .176 | .318 | .181 | — |

*Note*: *$p$<.05. **$p$<.01.

develop useful strategies. It appears that gifted children do not differ from their non-gifted peers when they are restricted from developing efficient strategies. On the other hand, when given opportunities, gifted children appear to quickly find useful strategies. Gifted children's ability to develop efficient strategies appears to separate them from their age peers in the domain of their strengths. However, this alone does not facilitate the development of working memory capacity. Without advanced development in working memory capacity, it is unlikely that the general rate of development is facilitated.

Another finding of interest is the relation between conceptual understanding and achievement. We found little or no relation between the number knowledge task and the SAT. When age was partialled out, the correlation coefficient was $-.021$ for this sample of children. This has implications for the selection of gifted children. The participants in this study scored higher on the average than their age peers on both these measures but there is little overlap in what each measure is assessing. Those children who scored below the 95th percentile on the SAT did show high levels of conceptual understanding of number. It is reasonable to assume that the SAT and number knowledge task measure different aspects of mathematical understandings. The number knowledge task may play a similar role to some aspects of available intelligence tests. This is an empirical question. It may prove worthwhile to consider the use of the number knowledge task as part of an identification battery for gifted children. It should also be mentioned that the SAT scores had little relation to the working memory measures, especially when age was partialled out. One theoretical implication of these findings is that mathematically gifted children have facilities that compensate for their age-related working memory constraints. We speculate that gifted children probably take advantage of their ability to develop strategies to efficiently solve mathematics problems. The current study highlights a complex relation that exists for gifted children among working memory, conceptual knowledge, and school performance in the mathematical domain.

## Study 2

### *Structural changes in gifted children's conceptual understanding in the mathematical domain*

In the first study, we found that mathematically gifted children in first and third grades were one to two years ahead of their age peers in the development of mathematical concepts but the rate of development of working memory was similar to their peers. Case's theory of intellectual development was used to explain the discrepancy in the two types of performance. That is, mathematically gifted children excel in the domain of their strengths but advanced conceptual understanding did not result in superior working memory capacity. This in turn places an age-typical upper bound at each stage of development, with gifted children no exception. In the second study, we focused on the development of mathematically gifted children's conceptual understanding of number. Using Case's theory as a framework, this study examined the changes in the organization of mathematical ideas in gifted children 5 to 6 years old.

*Development of central numerical structure*

Case postulated that children between the ages of 5 and 7 years old develop an important conceptual structure that allows them to benefit from formal learning of mathematics. This conceptual structure has been termed a 'central numerical structure' (CNS) and is believed to result from an integration of children's physical counting and visual comparisons of quantities. Research has documented that 4-year-old children typically are able to count a set of objects in a reliable fashion, and understand that the final number counted represents the cardinality of that set (Gelman, 1978). They have also been found to be able to visually compare quantities, determine one quantity to be more or less, and understand consequences of adding or taking away objects (Starkey, 1992). As yet, these two understandings have yet to be united in any coherent way. This type of understanding is characterized as 'predimensional' (or Level 1) thought.

Between the ages of 5 and 7 years old, an important development takes place. Children's physical counting and visual comparisons are no longer separate activities and they become aware of how the cultural convention of number words can index quantity differences. This achievement is characterized as the development of the 'unidimensional' (or Level 2) thought. This structural integration of counting and quantity schemas results in a central numerical structure that allows young children to reason with numbers (as opposed to a need to always rely on concrete objects or visual–spatial aids). Yet their understanding of numbers is limited to single digits or a group of single units. That is, even two-digit numbers such as 12 are thought of as a group of singles as opposed to a composite of tens and ones. The developmental milestone of understanding place value is characterized as 'bidimensional' (or Level 3) thought in the development of the central numerical structure. This understanding does not typically develop until about 8 to 9 years old. The next developmental milestone occurs around 10 to 11 years old. By then, children begin to understand numbers as consisting of multiple dimensions that are linked via explicit rules about numerical relations. This type of understanding is characterized as 'integrated bidimensional' (or Level 4) thought.

The current study used this developmental theory as a framework to examine the structural changes in mathematically gifted children. Analyses focused on gifted children under 7 years old who are expected by theory to be in the process of constructing a unidimensional conceptual numerical structure. We predicted that gifted children's advanced performance in mathematics is due to their earlier integration of the counting and quantity schemas to form a central numerical structure. Children of similar ages not identified as gifted formed a comparison group.

## *Method*

*Participants.* This study used data sets made available from two independently conducted studies—a study of mathematically precocious young children[5] and a study of non-gifted children.[6] The former study's sample included 310 incoming kindergarten and first grade children who were identified as mathematically gifted. These children were selected on the basis of scoring at the 98th percentile or above on at least one of two measures: arithmetic subtests of the K-ABC (Kaufman & Kaufman, 1983) and the WPPSI-R (Wechsler, 1989). The latter study's sample included 148 non-gifted children in kindergarten and first grade. In order to make the two samples comparable in age, we included only those children who were between the ages of 5 and 7 years old in the spring. The mean age of 6-years-old, or the age range of 5 to 7 years old, corresponds to the unidimensional substage of development in Case's theory. This resulted in 128 mathematically gifted children and 129 non-gifted children. Mathematically gifted children were tested once in late summer to fall and once again in the spring (approximately eight months later). Their mean ages were 5.5 years ($SD$=.31) in the fall and 6.1 years ($SD$=.32) in the spring. A group of non-gifted children was tested once in the spring; their mean age was 6.4 years ($SD$=.42).

*Measures and procedures.* Of the various measures administered to children, the number knowledge task mentioned in Study 1 was of interest in this study. This task has gone through several revisions and the gifted and non-gifted studies used different versions of the measure. For the purpose of conducting structural analyses, we included items that were given to both groups of children. Their performance on the full scale is also provided for comparison purposes.

The target items came from the unidimensional substage (Level 2) of the number knowledge task. These items were designed to measure children's integration of the counting and quantity comparison schemas. Five items were included in the analysis. These items were:

1. If you had four chocolates and someone gave you three more, how many chocolates would you have altogether?
2(a). What number comes right after seven?
2(b). What number comes two numbers after seven?
3(a). Which is bigger: five or four?
3(b). Which is bigger: seven or nine?
4(a). Which is smaller: eight or six?
4(b). Which is smaller: five or seven?
5. Which number is closer to five: six or two?[7]

Children had to answer correctly to both (a) and (b) questions in order to receive one point. The maximum possible score was five.

*Data analysis.* The primary focus of the analysis was to examine whether or not gifted children would undergo the integration of the counting and quantity comparison schemas into a central numerical structure in a similar fashion to, but at an earlier age than, typically developing children. Using structural equation modeling procedures based on the analysis of covariance, we compared the fit of one-factor and two-factor measurement models for the gifted data obtained in the fall and eight months later in the spring (see Figure 2). We also compared the fit of the two models for the non-gifted sample of children obtained in the spring.

We used the maximum likelihood (ML) estimation procedures for these analyses.[8] This method has been shown to be robust to violations of the multivariate normality assumption (McDonald & Ho, 2002). There were no missing data to require any form of replacement mechanism. Each measurement model was assessed for goodness of fit using multiple criteria; these were the $\chi^2$ likelihood ratio, the Comparative Fit Index (CFI; Bentler, 1990), the Tucker-Lewis Index (TLI; Tucker & Lewis, 1973), and the Root Mean Square Error of Approximation (RMSEA; Browne & Cudeck, 1993).

The $\chi^2$ value indicates the discrepancy between the observed covariance matrix and that estimated by the model relative to the degrees of freedom. A non-significant $\chi^2$ value indicates a fit of the data to the model. The significance level of this absolute index, however, is influenced by sample size. This dictates the power to find differences. The RMSEA is also a global fit index that takes into account model complexity (number of estimated parameters) within its calculation, favoring more parsimonious models. An RMSEA value of 0 indicates a perfect fit. It is suggested that RMSEA values less than .06 indicate a good fit and values less than .08 correspond with acceptable fit. Values beyond .08 suggest mediocre model fit (Hu & Bentler, 1999). The TLI is an incremental fit indicator that was initially developed for the purposes of comparing models of a factor analysis (Tucker & Lewis, 1973). This type-2 fit index typically ranges from 0 to 1 but can fall outside this range. Higher values indicate better fit. It has been considered convention for TLI values greater than .90 to show an acceptable fit. More stringent guidelines are proposed from simulation analyses that suggest TLI values greater than .95 to indicate an

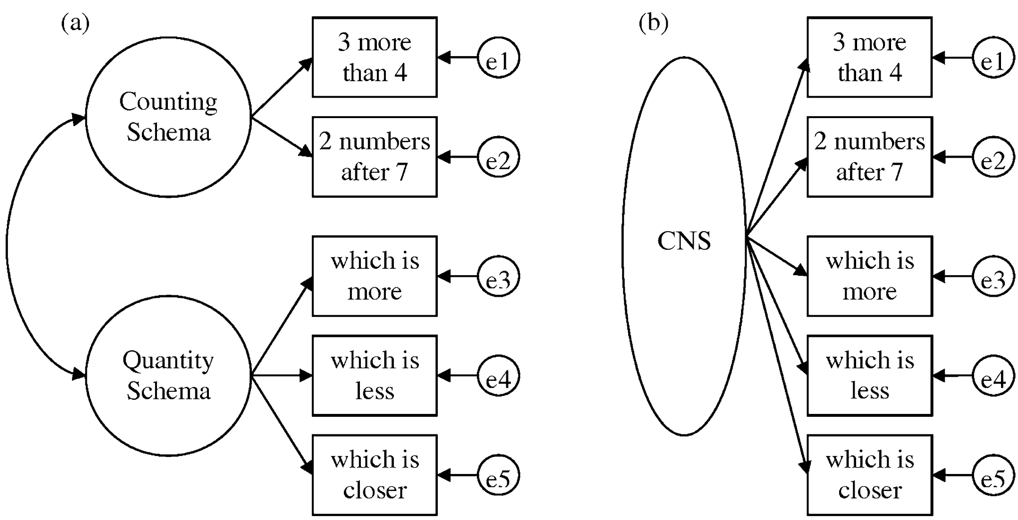


Figure 2. Measurement models to be tested: (a) two-factor model depicting two separate schemata, and (b) one-factor model depicting an integrated central numerical structure

acceptable fit (Hu & Bentler, 1999). The CFI is also an incremental fit index that rescales the $\chi^2$ value to range from 0 (no fit) to 1 (perfect fit) while adjusting for sample size. This type-3 indicator compares the specified factor model to a null model in which each observed variable is considered a separate factor. CFI values greater than .95 indicate a relatively good fit (Hu & Bentler, 1999).

### *Results*

*Overall performance on the number knowledge task.* As mentioned earlier, the two studies used different versions of the number knowledge task. The gifted study used a full version of the scale with a total of 36 items that assessed children's conceptual understanding from predimensional to the level beyond integrated bidimensional thought. The non-gifted study, on the other hand, used a short version of the scale with only 13 items covering predimensional to bidimensional thought. Our comparison of overall performance was therefore based on children's performance on the predimensional to bidimensional level items. Although this approach underestimated gifted children's actual performance, we found a large difference in children's spring performance on this task. The mean scores (standard deviation) for gifted children were 2.01 (.62) in the fall and 2.45 (.73) in the spring. It was 2.14 (.58) for non-gifted children in the spring. At the end of the school year, gifted children were roughly one year ahead of their non-gifted peers.

*Integration of the counting and quantity schemas to a central numerical structure.* The target items used for structural analyses were drawn from the unidimensional level. Children's performance on these items is listed in Table 3. Gifted children's performance in the spring was near perfect on these items with tighter standard deviations than those of non-gifted children. The comparisons of gifted and non-gifted children's performance on these items confirmed statistically significant differences in favor of gifted children, $t(255)=3.83$, $p<.001$, for the spring comparison (i.e., same testing time). We found through close inspection of performance differences between counting items and quantity items that children tended to do better on the quantity items (Table 4). This analysis also revealed that by spring gifted children did equally well on counting and quantity items whereas non-gifted children showed a discrepancy between the two types of items.

Table 3. Means (standard deviations) for the target items by group and testing time

| Item | Gifted fall | Gifted spring | Non-gifted spring |
|---|---|---|---|
| 3 more than 4 | .73 (.45) | .89 (.31) | .74 (.44) |
| 1 & 2 after 7 | .88 (.33) | .94 (.24) | .64 (.48) |
| Bigger | .92 (.27) | .95 (.21) | .95 (.23) |
| Smaller | .93 (.26) | .94 (.24) | .91 (.29) |
| Closer | .88 (.32) | .91 (.29) | .88 (.32) |
| Total | 4.34 (1.19) | 4.63 (1.12) | 4.12 (1.01) |

Table 4. Proportions of correct responses items for counting schema, quantity schema and total by group and testing time

| Item type | Gifted fall | Gifted spring | Non-gifted spring |
|---|---|---|---|
| Counting | .801 | .914 | .690 |
| Quantity | .911 | .932 | .912 |
| Total | .867 | .925 | .823 |

The data on the five items were used to examine the fit of one-factor and two-factor models. The covariance matrices, which served as the basis for the ML estimation, are presented in Tables 5 and 6. Preliminary analyses indicated some negative skew and positive kurtosis. Although the maximum likelihood method has been shown to be robust and relatively unaffected by violations of multivariate normality, we noted the potential for conservatively biased standard error terms.

Results from the factor analysis on the gifted fall data showed a good fit for both one-factor and two-factor models. Fit indices for a two-factor solution were a $\chi^2$ (4, $N$=128)=2.433, $p$=.657, RMSEA<.001, CFI=1.00, TLI=1.020. These statistics indicated a very good fit. In comparison, the single-factor model demonstrated a good but slightly less favorable fit than the two-factor model, $\chi^2$ (5, $N$=128)=4.492, $p$=.481, RMSEA<.001, CFI=1.00, TLI=1.005. An examination of the estimated correlation between the two factors in the two-factor model found a high correlation between the two factors ($r$=.851). Taken together, these

Table 5. Covariance matrices of the five items for the gifted sample at pre-test (below diagonal) and at post-test (above diagonal)

| Item | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1. 3 more than 4 | — | .048 | .042 | .048 | .045 |
| 2. 1 & 2 after 7 | .052 | — | .044 | .051 | .041 |
| 3. Bigger | .034 | .045 | — | .044 | .043 |
| 4. Smaller | .044 | .046 | .050 | — | .041 |
| 5. Closer | .039 | .040 | .038 | .039 | — |

Table 6. Covariance matrix of the five items for the non-gifted sample

| Item | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1. 3 more than 4 | — | | | | |
| 2. 1 & 2 after 7 | .038 | — | | | |
| 3. Bigger | .017 | .020 | — | | |
| 4. Smaller | .014 | .013 | .034 | — | |
| 5. Closer | .000 | −.011 | .017 | .036 | — |

results suggest that prior to the age of 6-years-old, gifted children were on the path to integrating the counting and quantity schemas. Factor loadings for both models are presented in Table 7.

At the end of the school year, gifted children's data showed the two-factor model's fit indices of $\chi^2$ (4, $N$=128)=13.318, $p$=.010, RMSEA=.135, CFI=.982, TLI=.955. The estimated correlation between the two factors was near perfect ($r$=.998). The single-factor model's fit indices were $\chi^2$ (5, $N$=128)=13.323, $p$=.021, RMSEA=.114, CFI=.984, TLI=.968. These suggest a more acceptable fit for a one-factor than two-factor solution.

As for the non-gifted sample, fit indices for a two-factor solution were, $\chi^2$ (4, $N$=129)=5.962, $p$=.202, RMSEA=.062, CFI=.969, TLI=.922. The estimated correlation between the two factors was $r$=.310. The fit indices for a one-factor solution were $\chi^2$ (5, $N$=129)=9.436, $p$<.093, RMSEA=.083, CFI=.929, TLI=.858. Although the $\chi^2$ value was non-significant for both solutions, the other indices showed a drop in fit from good fit in the two-factor solution to a mediocre fit based on RMSEA and an unacceptable fit based on TLI in the one-factor solution.

### *Discussion*

The primary interest of this study was to examine if mathematically gifted children achieved a conceptual integration of the counting and quantity schemas earlier than expected of their age peers. The current study found that gifted children under 6-years-old already showed signs of this integration and, eight months later, the development of the central numerical structure appeared to be in place. In comparison, non-gifted children 6-years-old were still in the process of integrating the two schemas. These differences in the rate of development in the mathematical domain explain superior performance of gifted children on various cognitive tests in the quantitative domain.

The current study also helped explain earlier findings by Robinson *et al.* (1996) who reported a clear distinction between quantitative and verbal abilities but a close

Table 7. Standardized factor loadings (regression weights) for two-factor and single-factor solutions

| | Gifted fall | | | Gifted spring | | | Non-gifted spring | | |
|---|---|---|---|---|---|---|---|---|---|
| Item | CNS | Count | Quant. | CNS | Count | Quant. | CNS | Count | Quant. |
| 3 more than 4 | .424 | .479 | | .684 | .685 | | .152 | .458 | |
| 1 & 2 after 7 | .634 | .733 | | .924 | .925 | | .132 | .391 | |
| Bigger | .804 | | .806 | .936 | | .936 | .604 | | .591 |
| Smaller | .880 | | .884 | .924 | | .924 | .854 | | .872 |
| Closer | .547 | | .545 | .672 | | .672 | .430 | | .428 |
| 2 factor $r$ | | .851 | | | .998 | | | .310 | |

association of quantitative to visual–spatial abilities among mathematically precocious young children. The current study used Case's theory that explained the close relation between quantitative and visual–spatial abilities as the root of later conceptual development in the mathematical domain. That is, an important conceptual development begins with children's understanding of counting (numerical) and quantity comparisons (visual–spatial), which are later integrated to become a unidimensional central numerical structure. This new structure no longer requires a heavy reliance on visual–spatial ability. Rather, it allows children to carry out numerical reasoning on the basis of numerical magnitudes. The current results from mathematically gifted children at two different times support this theoretical analysis by showing that the two-factor solution prior to 6-years-old and the one-factor solution eight months later can describe the development of their conceptual structure in the quantitative domain.

## General discussion

In this paper, we examined mathematical precocity in young children from a neo-Piagetian perspective. Two aspects of mathematical precocity were addressed in two separate studies. The first study examined mathematically gifted young children's working memory. The second study explored the conceptual integration of counting and quantity comparison schemas. The main findings from these studies were that mathematically gifted young children showed an earlier integration of important understandings in the numerical domain than did their age peers; the rate of development of working memory, however, was not different from their age peers.

In interpreting these findings, two interrelated ideas from Case's (1996) theory are particularly important. One is the distinction between domain-general and domain-specific development; and the other is the role of working memory in conceptual development. Case (1992; Case & Okamoto, 1996) postulated the notion of central conceptual structures as domain-specific semantic networks that consist of a unique set of meanings and rules. Therefore, conceptual growth is viewed as taking place in individual domains. That is, children could excel in one domain but not necessarily in others. However, system-wide constraints, such as working memory, set upper-limits on the rate of development across domains. Viewed from this perspective, it is reasonable to suggest that gifted and non-gifted children do not necessarily differ in their working memory capacity.

Case (1996) also stated that an increase in working memory capacity facilitates the acquisition of central conceptual structures, but not vice versa. If this assertion is correct, mathematically gifted children's advantages are less likely to arise from advanced working memory. Rather, their advantages are likely to result from how they use available working memory capacity to their advantage. We speculate that gifted children quickly develop efficient strategies to solve problems. Our data confirm that when their ability to generate efficient strategies is limited, gifted children do not demonstrate greater working memory capacity. Although a small sample size limits our ability to assert conclusive statements, this seems consistent with those who argue that deliberate practice (Ericsson, 2003; Ericsson *et al.*, 2005) may underlie intellectual advantages for gifted children. Case (1996) also stated that effective instruction in a particular domain facilitates children's conceptual development in that domain. Perhaps gifted children benefit from intellectual stimuli that serve as though they were receiving deliberate instruction. It may also be that gifted children's ability to rapidly devise efficient strategies allows them to focus on features of tasks at hand that must be highlighted for successful problem solving. This position, however, does not necessarily negate the idea of initial differences in brain organization between gifted and non-gifted children (O'Boyle *et al.*, 1991, 1994). It is possible that gifted children are born with atypical brain organization that allows them to benefit more from deliberate practice than those without such brain organization. A deeper understanding of gifted children's brain organization and strategy development is needed if educators are going to be able to capitalize on their early development of complex conceptual structures.

The current findings clearly point out the importance of examining gifted children's strategies. Without this focus, it is difficult to reconcile controversial findings where gifted children have sometimes demonstrated advantages over same age peers (Schofield & Ashman, 1987; Segalowitz *et al.*, 1992; Saccuzzo *et al.*, 1994), but other times have not (Globerson, 1985; Porath, 1992, 1996, 1997). Siegler's (2004) use of microgenetic methods and other approaches that look closely and carefully at what children do as they solve intellectual problems have the potential to enrich our understandings of conceptual development. These methods provide 'high density observations of learning' (Siegler, 2004, p. 364), which may be useful in understanding what strategies gifted children use in educational settings and how they are able to develop such highly efficient strategies when their age peers are not. Taken together with research demonstrating that children can benefit from explicit strategy instruction (Tournaki, 2003; Torbeyns *et al.*, 2005), we suggest gifted children's strategy development as a productive area of research, having theoretical implications for understanding characteristics of giftedness as well as practical implications for designing effective instruction for non-gifted children's development of efficient strategies.

Finally, the current paper highlighted the importance of examining structural changes in mathematically gifted children's conceptual development. Using Case's theory, we examined how gifted children's counting and quantity schemas are integrated to become a unified conceptual numerical structure. The current data showed that this integration takes place in gifted children roughly one year ahead of their age peers. An interesting aspect of this finding is the relation between counting (numerical) and quantity comparison (visual–spatial) schemas that are typically assembled prior to 5-years-old. The counting schema is presumed to have its origins in children's numerical discrimination and later physical counting, and the quantity comparison schema in children's visual–spatial abilities (Starkey *et al.*, 1990; Curtis, 2003). When these two schemas are integrated, children assemble a central numerical structure that takes numbers as the objects of mental manipulations (Okamoto, 1996). This structure relies considerably less on visual–spatial ability.

That is, the numerical domain becomes more clearly distinguishable from the visual–spatial domain of knowledge. Mathematically gifted children's earlier integration of the two schemas appears to contribute to the development of specialized ability in the numerical domain. Recent studies of brain imaging during simple calculation, however, point out the possibility of overlap between numerosity and visual–spatial working memory (see for example, Simon *et al.*, 2004). Continued effort by researchers is necessary to untangle relations between numerical and visual–spatial abilities, domain-specific and domain-general development, and mind and brain in gifted and non-gifted children.

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## Notes

1. This is equivalent to Case's 'central conceptual structure in the numerical domain'. Central conceptual structures are domain-specific in content but at the same time subject to system-wide constraints.
2. These measures require that participants carry out an action while trying to memorize target numbers or spatial layouts.
3. This system is typically described in Case's theory as consisting of four levels or sugstages: predimensional, undimensional, bidimensional and integrated bidimensional substages. These correspond to Levels 1 through 4, respectively.
4. This level is referred to as vectorial and includes rational numbers and negative numbers.
5. The gifted children participated in Project Math Trek (see Robinson *et al.*, 1996).
6. These children participated in the Case project (see Case & Okamoto, 1996).
7. Item 5 should have included two sub-items. However, a single item of this type was administered in the non-gifted study.
8. Because our items are dichotomous, the data lend themselves to violations of normality. Various methods have been developed to estimate from such data, for example, the categorical variable methodology (Muthén, 1984) and the asymptotically distribution-free (ADF) estimator (Browne, 1984). However, these estimation methods require a much larger sample size. We thus used the ML method.

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